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Research Article

Differential Equations and Applications to COVID-19

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Abstract

For this proposed research, a logistic growth model, specifically the Verhulst lo- gistic equation, developed by Pierre Francois Verhulst, is analyzed and solved using mathematical methods such as partial fraction decomposition and derivative rules. Its solution was anticipated to model the total COVID-19 cases in Texas, United States over time, and thus reveal possible future trends. However, due to the simplification of this logistic growth model, the results produced a somewhat accurate prediction.

Keywords: Verhulst Logistic Equation; Partial Fraction Decomposition; COVID-19

Introduction

Population dynamics, how population changes over time, can be observed using the logistic growth model. This model is a logistic function that can portray a population's growth rate and its change with limited resources in the environment. This specific model is discussed in various peer-reviewed original research papers, which were utilized in this paper. One research paper is written by A. Tsoularis, where it revealed the key features of this model [1]. Another research paper that utilized the logistic growth model is written by Huabin Wei, Yanqing Jiang, and Yuxing Zhang, where this equation is used to model the population growth in China [2]. Furthermore, COVID-19, caused by the severe acute respiratory syndrome coronavirus 2 virus, or SARS-CoV-2 for short, was first reported in officials in Wuhan City, China in December 2019 [3]. In close con- tact, can be spread through respiratory droplets. Because of this easy transmission, it resulted in the rapid spread of COVID-19 to many countries, causing hundreds of thousands of deaths around the globe, including the state of Texas in the United States [3]. Altogether, for this research paper, the number of COVID-19 cases in the state of Texas will be modeled using the logistic growth model and its solution to identify the current and future trends of COVID-19 cases. Because it views and analyzes the COVID-19 cases using a mathematical logistic growth model, the signifi- cance of this research is that, concerning the COVID-19 worldwide pandemic, it may introduce a different perspective on possible future trends of total COVID-19 cases in the Texas, United States. This allows potential practical applications such as determining policy actions that can influence the trend positively.

Objectives

Using advanced mathematical tools, the objective is to solve the Verhulst logistic equation and use its solution to determine future trends of COVID-19 cases in Texas, United States.

Hypothesis

If mathematical methods and tools are used to solve for the population in the logistic growth model, then it is expected that the solution can be used to represent the number of COVID-19 cases in Texas, United States accurately given the current trend and thus, the future trends as well.

Methodology

If mathematical methods and tools are used to solve for the population in the logistic growth model, then it is expected that the solution can be used to represent the number of COVID-19 cases in Texas, United States accurately given the current trend and thus, the future trends as well.

$$\frac{dP}{dt} = aP(1-\frac{P}{E}) \qquad (1)$$

Its general solution can be identified through the isolation of the variable P. Together with the COVID-19 cases data points, it will produce a logistic growth model that can model COVID-19 cases in Texas. To limit factors that may affect the number of cases given, the data points used in this research, obtained from the Texas Department of State Health Services, or DSHS, will have an initial date on March 24, 2020, where total daily case count includes "cases reported publicly by local health departments" instead of solely validated cases by the DSHS [4] [5]. This makes the succeeding total accumulated number of cases constant in terms of its source.

Finding General Solution

To solve this logistic growth model, the variable P will be isolated to produce an equation that shows the population given time, t. First, dt and $\left(1-\frac{P}{E}\right)$ is multiplied on both sides to isolate P on one side, the left side of the equation. Next, integrals are applied to both sides of the equation.

$$\int \frac{E}{P(E-P)} dP = \int a\ dt \qquad (2)$$

On the left-hand side, an additional simplification is required. Thus, partial fraction decomposition will be applied: each factor found in the denominator is written as a singular fraction with an unknown numerator. With these fractions added together, the same value in the denominator is multiplied on both sides to remove the fractions.

$$\frac{E}{P(E-P)} = \frac{A}{P} + \frac{B}{E-P}$$

$$P(E-P)\frac{E}{P(E-P)} = P(E-P)\frac{A}{P} - P(E-P)\frac{B}{E-P}$$

$$E = A(E-P) - BP$$

To identify the constants A and B, P and E will be given a value,

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specifically 0 and P respectively, to substitute in its variable allowing it to cancel out the other constant.

Let $P = 0$,

$E = A(E - 0) - B(0)$

$E = AE$

$A = 1$

Let $E = P$,

$P = A(P - P) - BP$

$P = -BP$

$B = 1$

These results above are substituted into the previous equation to produce:

$$\int \frac{1}{P} - \frac{1}{E-P} dP = \int a\ dt$$

Integrating,

$$\ln|P| - \ln|E-P| = at + C$$

$$\ln\left|\frac{P}{E-P}\right| = at + C$$

For some constant c.

Exponentiation and solving for P,

$$\frac{P}{E-P} = ce^{at}$$

$$P = \frac{ce^{at}E}{1+ce^{at}}$$

Finally multiplying the numerator and denominator by ce^{-at},

$$P = \frac{E}{ce^{-at}+1} \qquad (3)$$

With initial condition of $P(0) = P_0$ and solving for our constant, c,

$$P_0 = \frac{E}{c+1}$$

$$c = \frac{E-P_0}{P_0} \qquad (4)$$

Lastly, substituting (4) into the general solution (3),

$$P = \frac{E}{ce^{-at}+1}$$

$$P(t) = \frac{EP_0}{e^{-at}(E-P_0)+P_0} \qquad (5)$$

6. Data Analysis

The following is the application of the solution of the logistic growth model on COVID-19 cases in Texas, United States, where t represents the number of months since March 24, 2020, a is the relative growth rate, E is the carrying capacity of the number of cases, and P represents the total number of COVID-19 cases accumulated given the time as seen in equation (5).

Before the accumulated COVID-19 cases in Texas, United States can be modeled, first, the constants a and E will need to be identified. To do so, E is isolated from the equation, as it is easier to isolate compared to a. This is done by multiplying and distributing $(E-P_0)e^{-at}+P_0$, allowing the equation to rearrange.

$$P((E-P_0)e^{-at}+P_0) = EP_0$$

$$E = \frac{PP_0(e^{-at}-1)}{Pe^{-at}-P_0} \qquad (6)$$

Using this equation above, a system of the equation can be produced, where their intersections can identify the constants a and E. Specifically, while two equations share the same equation above, the values for t and P are varied. Thus, the following conditions are selected:

$t = 0$, $P_0 = 712$; where $t = 0$ represents March 24, 2020[4]

$t = 2$, $P_2 = 55,348$; where $t = 2$ represents May 24, 2020[4]

$t = 3$, $P_0 = 125,921$; where $t = 3$ represents June 24, 2020[4]

With the selected conditions, the two equations can be written as:

$$E_2 = \frac{55,348(712)(e^{-2a}-1)}{55,348(-712)e^{-2a}}$$

$$E_3 = \frac{125,921(712)(e^{-3a}-1)}{125,921(-712)e^{-3a}}$$

Next, these equations are inserted into a graphing display calculator, specifically a TI-84 plus CE calculator, where E_2 and E_3 are represented by both Y_1 and Y_2 respectively, and the constant a is represented as x, shown in figure 1. By graphing these two equations, the intersection reveals the solutions to the unknown variables of E and a.

From the graphing display calculator, shown in figure 2, the intersection is located at (2.416971, 143,887.12). Thus, it suggests that $a = 2.416971$ and $E = 143887.12$, where it is the relative growth rate and the carrying capacity of the number of COVID-19 cases respectively. These values are substituted into the solution of the logistic growth model, producing the mathematical model of COVID-19 cases in Texas overtime, represented as P_g:

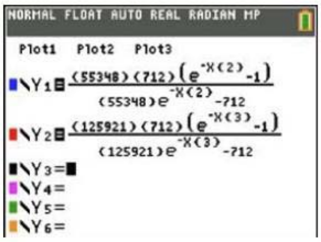
$$P_g = \frac{102,447,629.40}{(143175.12)e^{-2.416971t}+712}$$

Additionally, it can also be modeled beyond the given time, for example, when $t \to \infty$. Using the solutions of the logistic growth model, this model can be written as a limit, where $a > 0$:

$$\lim_{x\to\infty} P = \frac{EP_0}{(E-P_0)e^{-at}+P_0}$$

In this equation, at t approaches to infinity, e^{-at} approaches zero, given that the horizontal asymptote of $y = e^{-x}$ is $x = 0$; thus $e^{-at} \to 0$:

$$\lim_{x\to\infty} e^{-at} = 0$$



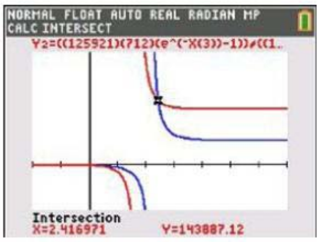


Figure 1 (on the left): Equations E2 and E3 are represented by both Y1 and Y2 and constant a is represented as x in the TI-84 Plus CE graphing calculator.

Figure 2 (on the right): Equations E2 and are graphed, where it is represented as blue and red lines respectively.

Letters represented on the graph	t Representing the number of months since March 24, 2020	P Representing the total number of COVID-19 cases accumulated given the time
A	0	712
B	0.5	10230
C	1	22806
D	1.5	37860
E	2	55348
F	2.5	77253
G	3	125921

Figure 3: The data points for t = 0.5, 1.5, 2.5 are represented as April 9, May 9, and June 9 respectively, where it is the estimated half point between each month.

Therefore,

$$\lim_{x\to\infty} P = E$$

This shows that as t approaches to infinity, P, the population of the given time, is equal to E, the carrying capacity of the population. In other words, in this model, the population, the COVID-19 cases in Texas, will ultimately reach its carrying capacity, the value of E [1].

Results

This equation, where a and E are substituted into the solution of the Logistic Growth model, is displayed on a graph with the actual COVID-19 data points, shown in figure 3. In the graph, t is found on the x-axis, and P is found on the y-axis. The actual COVID-19 data points used in the graph are shown in the chart in figure 4. This is done by using the online software GeoGebra Graphing Calculator. Due to the limits of the software, this equation is represented as g.

$$P_g = \frac{102,447,629.40}{143175.12e^{-2.416971t} + 712} \quad (7)$$

Discussion

Overall, the results in the graph above reveal the model fits the actual data moderately well. Because the mathematical model P_g, shown as g in the graph, is developed through the use of the data points A, E, and G, these data points are located on the model very well, shown in figure 5. However, the surrounding data points, B, C, D, and F fit onto the model to a moderate extent, shown in figure 5. This strongly suggests that the logistic growth model does not take into account other data points. Therefore, it fails to model the increasing trend in the actual COVID-19 data to a great extent.

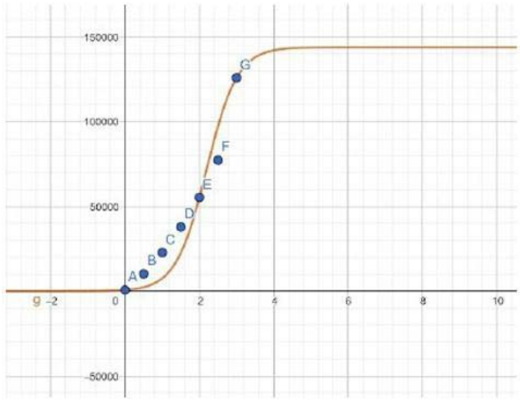


Figure 4: The graph of the mathematical model of COVID-19 cases in Texas overtime, where t is found on the x-axis and P is found on the y-axis.

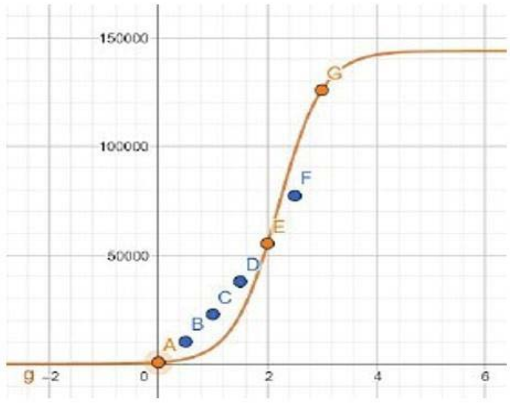


Figure 5: The graph of the mathematical model of COVID-19 cases in Texas overtime, where t is found on the x-axis and P is found on the y-axis. Data points A, E, and G are colored yellow to portray its location relative to the model.

$$P_g = \frac{102,447,629.40}{143175.12e^{-2.416971t} + 712}$$

This finding is supported by a peer-reviewed paper written by Huabin Wei, Yanqing Jiang, and Yuxing Zhang. When they attempt to model the population of China using the logistic growth model, they discovered that the model "takes t as the only explanatory variable", and that in real life "population growth, the growth rate r will not be constant" [2]. Applying their findings to this research, it suggests that outside factors, in this case, possible changes in government regulations and human movement, can affect the actual COVID-19 cases data that the model fails to consider [2]. Altogether, with the given data, this shows that this logistic growth model can model the current trends of COVID-19 accurately to a moderate extent, where the model suggests that in about 4 months from March 24, 2020, COVID-19 cases has nearly reached its carrying capacity at about 140, 000 cases, and the daily cases will drop. If this trend continues indefinitely, it will result in a total of about 143,887 cases. Nevertheless, as this is a current, ongoing pandemic, this exact value in the result is only a suggestion to future cases. Realistically, this model may only reveal small and limited predictions in the future. Because of uncertainties, this model may be beneficial only in implementing immediate, short-term plans of action to address COVID-19.

Conclusion

In this research paper, the logistic growth model, developed by Pierre Franc¸ois Verhulst, is ana- lyzed and solved through a series of arithmetic operations, differentiation of derivatives, and partial fraction decomposition. This transforms the logistic growth model from equation (1) to (5). This solution is then used to model cases of COVID-19 in Texas overtime since March 24, 2020, where the data points are obtained through the Texas Department of State Health Services website. Using three data points, it produced a mathematical model of COVID-19 cases in Texas:

$$P_g = \frac{102,447,629.40}{143175.12e^{-2.416971t} + 712}$$

Through this equation, on a graph, it reveals this model moderately models the actual data points. This is due to both the use of limited data points in the actual construction of the mathematical model, and possible outside factors that were not accounted for mathematically. Thus, while the model suggests that there will be a decrease of cases a few months after March 24, 2020, it is also possible for the number of COVID-19 cases in the future to vary; the exact value predicted as the maximum, 143,887 cases, is subject to change. Therefore, it suggests that the model may be only helpful in implementing short-term plans of action near the model's time range, approximately from March to July of 2020.

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