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Mini Review

Practical Calculation of the Leakance Coefficient to Estimate Seepage in Large-Scale Hydrologic Models: Examples and Graphical Illustrations

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Abstract

In most integrated hydrologic models the seepage from the rivers is evaluated from an empirical parameter, the "leakance coefficient", ʌ, which is estimated by calibration. It is assumed that this parameter remained the same throughout the historic period whether e.g. the flow in the river was high or low. Systematic research has shown that this leakance could be estimated theoretically. It was done through a combination of techniques largely analytical but also extensively numerical using very fine grids. A Stream Aquifer Flow Exchange (SAFE) dimensionless conductance, г, was derived. It is function of many factors: (1) the normalized wetted perimeter of the river cross-section, (2) the degree of penetration of the river into the aquifer, (3) the degree of anisotropy of the aquifer, (4) the size of the grid for the aquifer cell that contains the river and (5) the potential presence of a clogging layer in the riverbed. The relation between the leakance coefficient and the dimensionless conductance was derived. The steps necessary to calculate the SAFE dimensionless conductance as a function of all those factors are provided. Vice versa given values of a calibrated leakance one can assess how much of that leakance is due to an actual presence of a clogging layer or due to other factors. Many figures illustrate in turns how each factor influences the value of the dimensionless conductance.

Introduction

A systematic effort over a decade was carried out as a team and published [1-7] with the goal to describe accurately the influence of the many factors that affect the value of the (one-sided) SAFE dimensionless conductance, г, and thus the leakance coefficient, ʌ. It is the purpose of this article to show how this research translates into a comprehensive and practical tool to help in the estimation of the seepage from a river in hydraulic connection with the aquifer.

Relation between the Leakance Coefficient and the SAFE Dimensionless Conductance

Typically, e.g. in the code MODFLOW, the seepage discharge is estimated as

$$Q_S^{\text{mod}} = C_{riv}(h_S - h_f) = \Lambda L_R W_p (h_S - h_f) \quad (1)$$

where h_s is the head in the river, h_f is the head at the node in the cell underlying the river reach (i.e the aquifer cell that contains the river reach, denoted the river cell) and C_{riv} is the hydraulic conductance of the river-aquifer interconnection (L^2T^{-1} i.e. dimension of a transmissivity). The parameter C_{riv} is itself estimated as $C_{riv} = \Lambda L_R W_p = \frac{K_{cl} L_R W_p}{e_{cl}}$ (2)

and the leakance coefficient ʌ is usually estimated as the ratio $\frac{K_{cl}}{e_{cl}}$ where K_{cl} is the hydraulic conductivity of the riverbed material (presumed to be a clogging layer), e_{cl} is its thickness, L_R is the length of the river reach at it crosses the node (that is the length within the aquifer cell that contains the reach, the river cell), and W_p is the wetted perimeter of the river reach.

With the SAFE dimensionless conductance approach the seepage discharge is defined as: $Q_S = 2 L_R K_H \Gamma (h_s - h_f)$ (3)

г is the (one-sided) SAFE dimensionless conductance and thus the reason for the factor 2 in the formula as Q_S is the full seepage discharge. K_H is the horizontal conductivity of the aquifer.

Identification of Equations. (1) and (3) yields: $\Lambda = \frac{2K_H}{W_p}\Gamma$ (4)

If г is known so is ʌ and thus C_{riv} as $C_{riv} = 2 L_R K_H \Gamma$ (5)

Formulae Needed to Evaluate and Immediate Use for a Set of Parameters

Later on in the article a number of figures show very generally the influence of the degree of penetration, of anisotropy, of the grid size and of the presence of a clogging layer within the riverbed. However in practice the user deals with a specific wetted perimeter, a specific aquifer thickness, a given degree of penetration, a given amount of anisotropy, a particular grid size and eventually a clogging layer in the river bed. The user wants to estimate not in general but for these particular values. For this reason as soon as the formulae are introduced the numerical example is provided. Figure 1 shows schematically a river cross-section and defines the variables that will affect the value of the conductance.

Given set of parameters

All values are expressed in meters (m) and rates are in per day. The river (creek) bottom half-with B = 2m, the river stage is H = 2m and the wetted perimeter is W_p = 2(B + H) = 8 m. (If the river cross-section is not rectangular, the usual situation, an equivalent

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rectangular cross-section is one with the same maximum depth, which defines the head in the stream, and the same wetted perimeter, which conditions the area through which seepage takes place).

Degree of penetration of the cross-section of the aquifer is defined as the ratio of the river stage H over the aquifer thickness, $\bar{D}_{aq}$, $d_P = \frac{H}{\bar{D}_{aq}} = \frac{2}{20} = 0.1$ (6). Similarly normalized wetted perimeter is defined as the ratio of the wetted perimeter divided by the aquifer thickness, $W_P^N = \frac{W_p}{\bar{D}_{aq}} = \frac{8}{20} = 0.4$ (7). For ease of use the results obtained analytically have been curve fitted by simple second order algebraic equations and the coefficients are tabulated in Appendix 1 for different ranges of normalized wetted perimeters and degrees of penetration for the case that the aquifer is isotropic and that the far distance away from the river bank is the precise minimum far distance from the bank at which the flow has become horizontal, which is conservatively estimated at $2\bar{D}_{aq}$ (in the case of isotropy). Figure 2 shows why that value is conservative.

In the case of Figure 2 the distance from the river bank to the outer boundary was chosen to be precisely twice the aquifer thickness. It is very clear that the flow has become essentially horizontal much before it reaches the outer boundary, in fact practically half way to the outer boundary. Figure 2 shows also that the flow is not horizontal in the vicinity of the river and is not purely vertical to the center of the river cell.

The chosen grid size for the study under consideration is G = 600 m.

The horizontal conductivity is estimated as K_H = 2.0 m/day

The anisotropy ratio $R_{anis} = \frac{K_V}{K_H}$ is estimated as 0.1.

The parameters of the clogging layer are estimated as conductivity K_{cl} = 0.1 m/day and thickness e_{cl} = 0.30 m, thus $\Lambda_{cl} = \frac{K_{cl}}{e_{cl}} = \frac{0.1}{0.3} = 0.333$.

Influence of degree of penetration in the case of isotropy

The value of ｒ in this case is: $\Gamma_{iso} = \Gamma_{flat}[1 + a_1 d_P + a_2 (d_P)^2]$ (8)

or numerically using the Table in Appendix 1:

$$\Gamma_{iso} = \Gamma_{flat}[1 + 0.89(0.1) - 2.43(0.1)^2] = 1.065\Gamma_{flat} \quad (8a)$$

To calculate Γ_{flat} one must first define the function κ of the normalized wetted perimeter: $\kappa = e^{-\frac{\pi W_p^N}{2}} = e^{-\frac{3.14159(0.4)}{2}} = 0.5335$ (9)

and $\Gamma_{flat} = \frac{1}{2[1 + \frac{1}{\pi}\ln(\frac{2}{1-\kappa})]} = \frac{1}{2[1 + \frac{1}{3.14159}\ln(\frac{2}{1-0.5335})} = 0.342$ (10)

Γ_{flat} is the value of Γ_{iso} when the degree of penetration is zero, thus the river cross-section is flat. Substitution of that value in Equation. (8a) yields:

$$\Gamma_{iso} = 1.065 * 0.342 = 0.364 \quad (8b)$$

(When the normalized wetted perimeter tends to infinity (extremely wide river) with a perfectly flat cross-section the SAFE dimensionless conductance (in short the conductance) reaches its maximum value as: $\Gamma_{flat} = \frac{1}{2[1 + \frac{1}{\pi}\ln(\frac{2}{1})]} = 0.41$).

However the aquifer is not isotropic. So the value of 0.364 must be corrected for anisotropy of the aquifer.

Influence of the degree of anisotropy on

If there is anisotropy naturally the results obtained assuming isotropy must be corrected. This requires a two-step process. First the value even under isotropy must now be evaluated at a minimum far distance from the river bank corresponding to the degree of anisotropy. That minimum far distance is $\frac{2\bar{D}_{aq}}{\sqrt{R_{anis}}} = \frac{2(20)}{\sqrt{0.1}} = \frac{40}{0.316} = 127$ which is greater than 80 m. The excess distance from the isotropic value normalized by the aquifer thickness is:

$$\Delta^*_{anis} = \frac{\Delta_{anis}}{\bar{D}_{aq}} = 2(\frac{1}{\sqrt{R_{anis}}} - 1) = 2(\frac{1}{0.316} - 1) = 4.329 \quad (12)$$ and

$$\Gamma_{iso-\Delta_{anis}} = \frac{\Gamma_{iso}}{1 + \Gamma_{iso}\Delta^*_{anis}} = \frac{0.364}{1 + 0.364(4.329)} = 0.1413 \quad (13)$$

Once ｒ has been evaluated at that proper anisotropic distance it must be adjusted for the proper degree of anisotropy.

Analytical results and numerical simulations have defined a reduction factor due to anisotropy as a function of the variable: $\xi = (1 - \sqrt{d_P})(1 - \sqrt{R_{anis}}) = (1 - \sqrt{0.1})(1 - 0.316) = 0.4675$ (14) and

$$R_f = 1 - 0.333\xi - 0.294(\xi)^2 = 1 - 0.333(0.4675) - 0.294(0.4675)^2 = 0.78$$

(15).

Clearly there is no reduction when there is isotropy (ξ = 0) and no reduction if full penetration is assumed (ξ = 0) as in this case the flow from the river is immediately horizontal, regardless of the degree of anisotropy.

Finally $\Gamma_{anis} = R_f \Gamma_{iso-\Delta_{anis}} = 0.78(0.1413) = 0.1102$ (16).

This is the value of the conductance at a distance from the river bank which is 127 m. But (see Figure 1) this is not the distance at which we want to evaluate the conductance but the distance $\frac{G}{4} - B$. An excess distance needs to be calculated.

Influence of the grid size on ｒ

That excess is: $\Delta G = (\frac{G}{4} - B) - \frac{2\bar{D}_{aq}}{\sqrt{R_{anis}}} = (\frac{600}{4} - 2) - \frac{2(20)}{0.316} = 21.4$ m (17)

The corrected value of the conductance is:

$$\Gamma_{anis-\Delta G} = \frac{\Gamma_{anis}}{1 + \Gamma_{anis}\frac{\Delta G}{\bar{D}_{aq}}} = \frac{0.1102}{1 + 0.1102(\frac{21.4}{20})} = 0.092 \quad (18)$$

Now there may be a clogging layer presence in the riverbed. An additional correction is needed.

Influence of the clogging layer on ｒ

$$\Gamma_{anis-\Delta G-cl} = \frac{\Gamma_{anis-\Delta G}}{1 + \Gamma_{anis-\Delta G}\frac{K_H}{K_{cl}}\frac{e_{cl}}{(B+H)}} = \frac{0.092}{1 + 0.092(\frac{2.0}{0.1})(\frac{0.3}{(2+2)})} = 0.0838 \quad (19)$$

Summary of the numerical example

Accounting for penetration the first (incomplete) estimate of the conductance was 0.364. Correcting for anisotropy the value was then

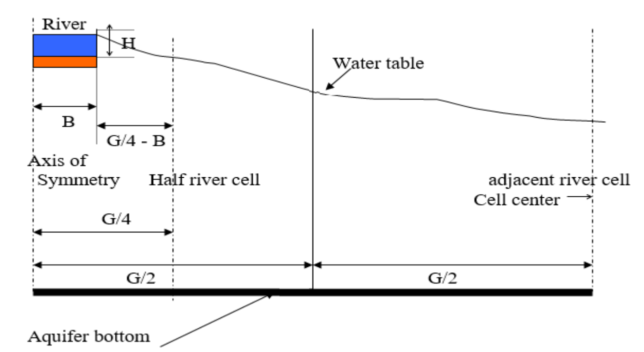

Figure 1: Schematic cross-section of river and aquifer cells. Not to scale.

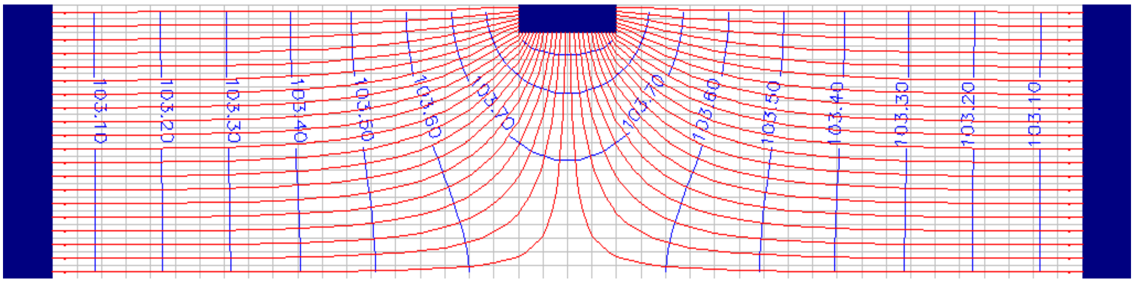

Figure 2: Exact analytical flow pattern from a rectangular cross-section with a moderate degree of penetration [3,4].

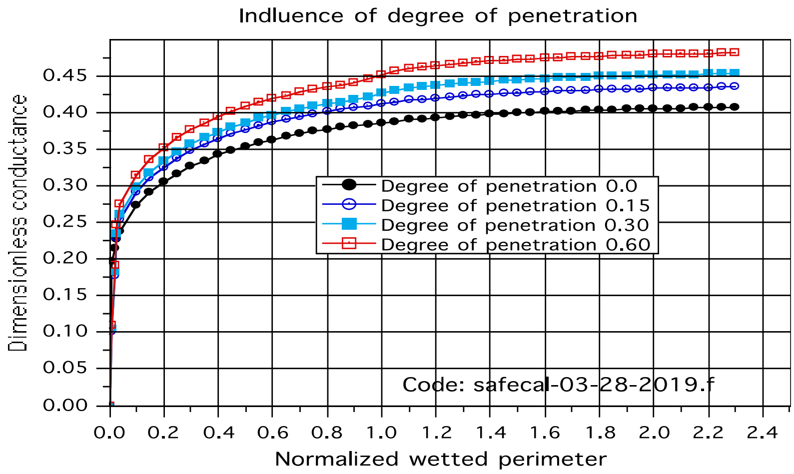

Figure 3: Conductance increases with penetration.

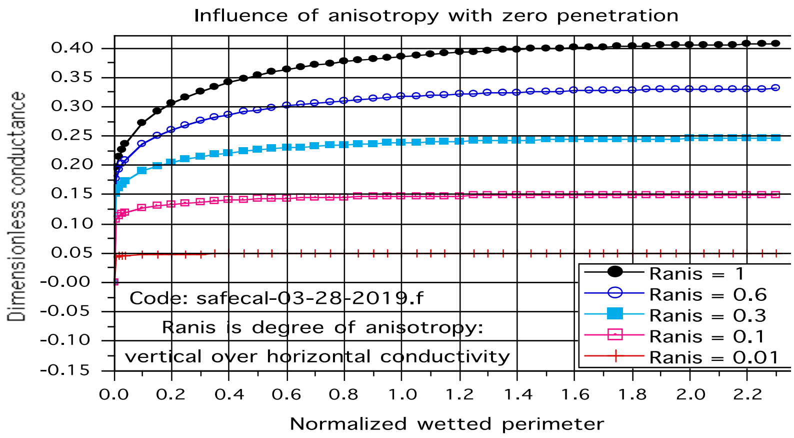

Figure 4: Conductance decreases with anisotropy.

adjusted to 0.110. Making the adjustment for the grid size it became 0.092 and finally including the resistance due to the clogging layer it was 0.084.

Influence of Each Individual Factor on the Conductance

In the previous sections the use of the formulae to estimate the conductance was done for a given set of parameters. Now we are looking to those influences one parameter at a time and display them graphically.

Influence of degree of penetration on the conductance

Figure 3 shows the influence of the degree of penetration on the conductance as a function of the normalized wetted perimeter, (1) assuming isotropy, (2) when the distance to the half-river cell center from the bank is exactly the minimum isotropic far distance and (3) there is no clogging layer.

Even for a very high value of the normalized wetted perimeter unless there is a high degree of penetration the conductance will not reach the value 0.5 corresponding to full penetration. A value of 0.6 for degree of penetration is already a very high value and probably

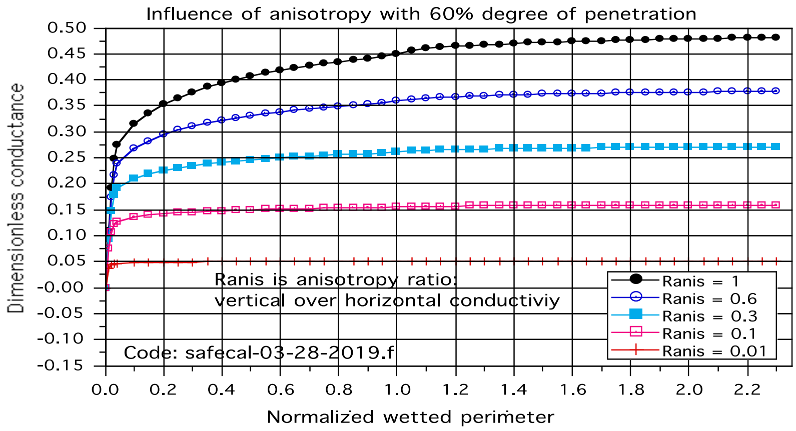

Figure 5: Penetration reduces the decreasing influence of anisotropy.

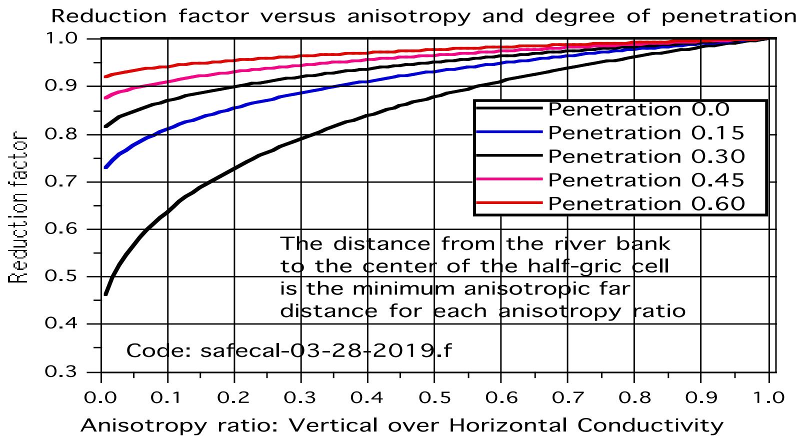

Figure 6: Reduction factor as a function of anisotropy and penetration.

not many rivers ever attain such a high degree of penetration. When full penetration is assumed [as in 8] the impact of a pumping well on river depletion is calculated the same way whether the creek is a few meters wide or the river is a hundred meters wide. Figure 3 shows vividly that this is incorrect.

Influence of the degree of anisotropy on r

Figure 4 shows the influence of anisotropy when the degree of penetration is zero, when the distance to the half-river cell center from the bank is exactly the minimum anisotropic far distance and there is no clogging in the riverbed.

When there is anisotropy even a moderate level such as a ratio of 0.6 will cause a significant decrease in the value of the conductance.

On the other hand as shown in Figure 5 when there is a significant degree of penetration the effect of anisotropy is much reduced because a good part of the seepage is taking place through the sides of the river rather than its bottom.

These figures do not provide a very useful comparison because for a given value of the normalized wetted perimeter the read values for different anisotropy ratios do not correspond to the same grid size. For the same grid size the comparison needs to be between $\Gamma_{iso-\Delta_{anis}}$ and Γ_{anis} but that is exactly R_f, the reduction factor. For instance for different values of the degree of penetration one can plot that reduction factor as a function of the degree of anisotropy. This is shown in Figure 6.

Influence of the grid size on r

Figure 7 shows the influence of the grid size on the value of the conductance with no penetration and isotropic conditions. If the conductance (i.e. leakance coefficient) was calibrated with a numerical model using a particular mesh it is clear that the calibrated value could not be used with a different mesh, without some correction. Figure 8 shows that the conductance value is also much affected by the grid size when there is a significant degree of anisotropy.

Influence of the presence of a clogging layer on r

Figure 9 shows the influence of a clogging layer when there is no aquifer penetration and isotropy. If there is no clogging layer the leakance of the riverbed is infinite. Figure 10 shows the influence of a clogging layer when there is no aquifer penetration and anisotropy. The decrease is amplified.

Discussion

The calculations of the SAFE dimensionless conductance, r, and consequently of the leakance coefficient, $\Lambda = \frac{2K_H\Gamma}{W_p}$, and of the parameter $C_{riv} = 2L_R K_H \Gamma$ can be performed with a simple subroutine

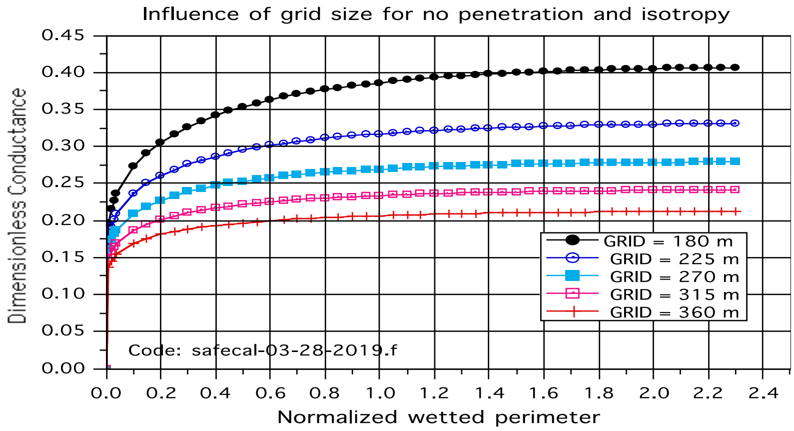

Figure 7: Grid size influences the value of the conductance.

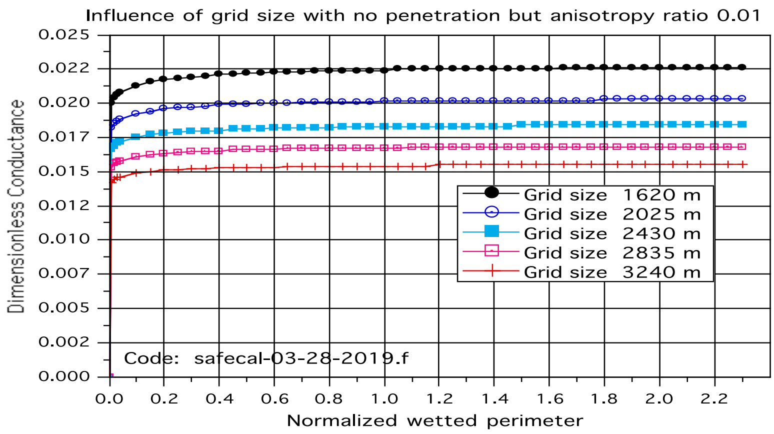

Figure 8: Strong anisotropy reduces greatly the influence of grid size on the conductance value.

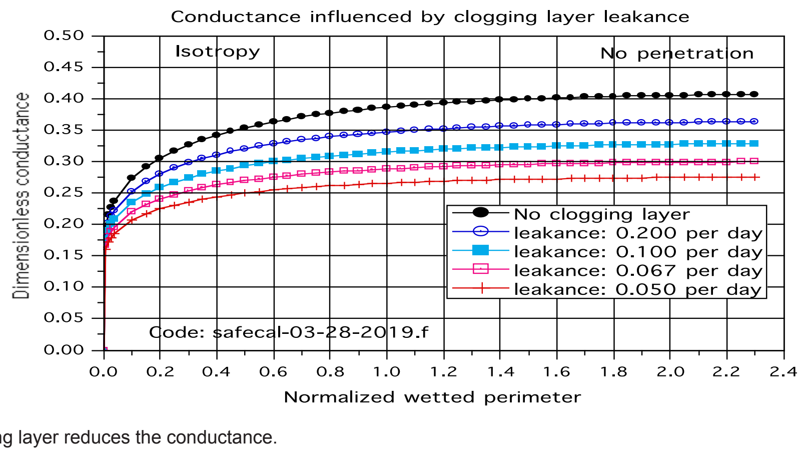

Figure 9: Presence of a clogging layer reduces the conductance.

within a hydrologic model.

Naturally to calculate r one must know about the geometry of the river cross-section (say B and H, more generally W_p) and the aquifer thickness, $\bar{D}_{aq}$. These are usually fairly well known. The grid size is known. (For a numerical example we use the values of section 3) More difficult is the estimation of the horizontal conductivity, K_H, of the degree of anisotropy and of the clogging layer parameters. Typically the values of K_H and of Λ are obtained by calibration as if these two parameters were independent. Let us assume that the calibration was carried out over periods of low flow so that the average value of W_p is reasonably well known and the degree of penetration can be considered negligible. Given the calibrated values of Λ = 0.08 per day and K_H = 2.0 m/day then $\Gamma_{cal} = \frac{\Lambda W_p}{2K_H} = \frac{(0.08)4}{2(2)} = 0.08$ (20).

First we assume that there was no anisotropy and no clogging

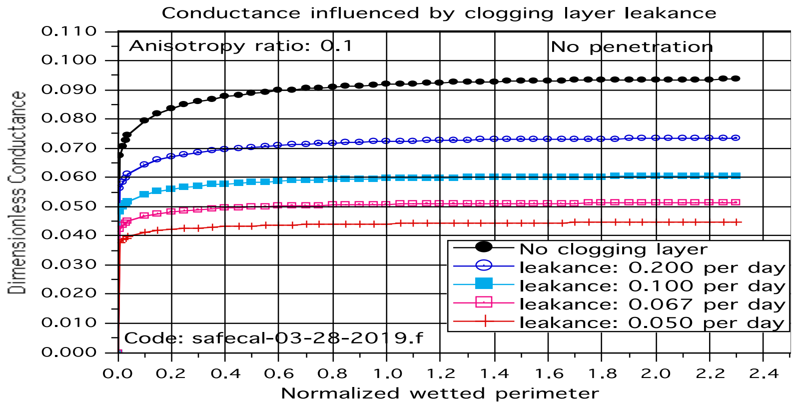

Figure 10: Decrease of the conductance due to the presence of a clogging layer is amplified by anisotropy.

layer. $W_p^N = \frac{W_p}{\bar{D}_{aq}} = \frac{4}{20} = 0.2$. Next evaluate the value of Γ_{flat}

First calculate the value of $\kappa = e^{-\frac{\pi W_p^N}{2}} = e^{-\frac{3.14159(0.2)}{2}} = 0.730$ then

$$\Gamma_{flat} = \frac{1}{2[1+\frac{1}{\pi}\ln(\frac{2}{1-\kappa})]} = \frac{1}{2[1+\frac{1}{3.14159}\ln(\frac{2}{1-0.730})} = 0.305$$

Next this must be evaluated at the proper distance to the center of the half-river cell. $\Gamma_{iso-\Delta G} = \frac{\Gamma_{iso}}{1+\Gamma_{iso}\frac{\Delta G}{\bar{D}_{aq}}} = \frac{0.305}{1+0.305\frac{108}{20}} = 0.115$ (21) with

$$\Delta G = \frac{G}{4} - B - 2\bar{D}_{aq} = \frac{600}{4} - 2 - 2(20) = 108 \quad (22)$$

The value of 0.115 exceeds the calibrated value of 0.08. The inference is that there is either some degree of anisotropy or a clogging layer. Let us assume that the consensus of the team carrying the investigation is that it is most likely the presence of a clogging layer rather than anisotropy.

The formula to use in reverse is:

$$\Gamma_{iso-\Delta G-cl} = \frac{\Gamma_{iso-\Delta G}}{1+\Gamma_{iso-\Delta G}\frac{K_H}{K_{cl}}\frac{e_{cl}}{(B+H)}} = \frac{0.115}{1+0.115(\frac{2.0}{K_{cl}})(\frac{e_{cl}}{(2)})} = \Gamma_{cal} = 0.08$$

(23) Solution gives $\Delta_{cl} = \frac{K_{cl}}{e_{cl}} = 0.263$ per day. This is much larger than the calibrated value $\Lambda_{cal} = 0.08$ per day. The implication is that the clogging layer contributed only or 30% of the resistance to seepage. 70% was due to the resistance to turn in the vicinity of the river bottom from a vertical direction to the horizontal one.

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