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Mini Review

Validating the Safe Dimensionless Conductance Approach to Estimate Seepage for a Sinuous River

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Abstract

In some fairly recent articles a new approach to estimate the flow exchange between a river and a connected aquifer was presented. It accounted for many factors such as normalized wetted perimeter, degree of penetration of the river into the aquifer, extent of anisotropy in the aquifer and the possible presence of a clogging layer within the streambed. That approach was developed primarily for use in studies for large regional areas. It has the advantage that it provides great accuracy while considerably reducing the numerical work. The derivations for the analytical solutions may give the superficial impression that the approach is mostly valid if the river channel is straight. The purpose of this paper is to demonstrate through case studies that the approach is valid even if the river changes direction significantly. To prove that point it is necessary first to discuss through several examples how typical numerical groundwater models calculate the seepage amount and how the values depend upon the level of discretization. Then the case of a river with a right angle turn is examined both using a standard numerical finite difference model with an extremely fine level of discretization and the SAFE approach. The study shows that the two approaches yield essentially identical results.

Keywords: Stream-aquifer interaction; SAFE dimensionless conductance; Impact of river sinuosity on seepage

Introduction

In previous articles an approach to estimate the saturated flow exchange between a river and an aquifer was presented [1-4]. It is meant as an alternative analytical procedure to costly and time-consuming numerical techniques requiring fine grids in order to maintain accuracy in large scale regional studies [2,3]. Once understood and accepted the procedure could replace the way many current groundwater models estimate the flow exchange [5-11].

As derived, the discharge is calculated for the flow in a vertical cross-section normal to the direction of the river. Thus, it would appear superficially that the solution might be valid only in the case of a straight river. Figure 1 shows a typical such cross-section showing the flow lines and the potential lines in a vertical cross-section. Figure 2 illustrates how a stream's actual channel might be represented in a numerical ground water model. It should be noted that the numerical models do not specify a direction for the river segment(s) within the cell. They only specify that there is river inflow on one side and outflow from another side. To avoid the complications resulting from the flow in the river and the drop in elevation of the streambed in the direction of flow, we investigate the steady-state flow exchange for a trench, even though we commonly use the word river to describe it, as the results would apply to a river as well.

Understanding How Most Numerical Models e.g. MODFLOW Calculate the Flow Exchange

The purpose of the first 3 runs is to show how the division of a stream reach into a variable number of segments affects the distribution of the seepage in the lateral versus the longitudinal direction.[12]

First case (first run; Figure 3): In a first case (first run) the entire trench is located within a single Finite Difference (FD) cell as shown in Figure 3. There is one geologic layer, not subdivided into multiple calculation sublayers in the vertical direction. The head in the trench is 120 (units of length; these are left unspecified. They could be feet or meters or whatever). The trench is shown in red. The aquifer thickness is uniformly 100. The cells are square with sides equal to 1000.

The width of the trench is 10 and its length is 1000. There is a clogging layer of thickness 0.5 and hydraulic conductivity of 10 per time period (e.g. day). The aquifer is isotropic and its conductivity is homogeneous of value 100. The boundaries of the aquifer are shown in black with uniform third type boundary condition with a value of the coefficient being high, 1000, just so it will take a small head difference in each boundary cell to transmit its share of the seepage. The external head at the boundaries is uniform at a value 110. Given that it is a steady-state condition the entire seepage flow must exit through the aquifer boundaries. Let Qs be the total seepage discharge from the trench and Qij the discharge into each of the four adjacent cells to the trench cell with i being the row index and j the column index. Using the MODFLOW code the total discharge Qs was calculated to be 94,032 with one fourth, 23,508, going into each adjacent cell. Note that in Figure 3 the direction of the trench is NS. However in MODFLOW the orientation of the trench does not matter as is clearly shown by the fact that the flow exchange with the four adjacent cells is the same. The ratio of seepage to the NS versus EW is: 1.0.

Second case (second run; Figure 4): In this second case (second

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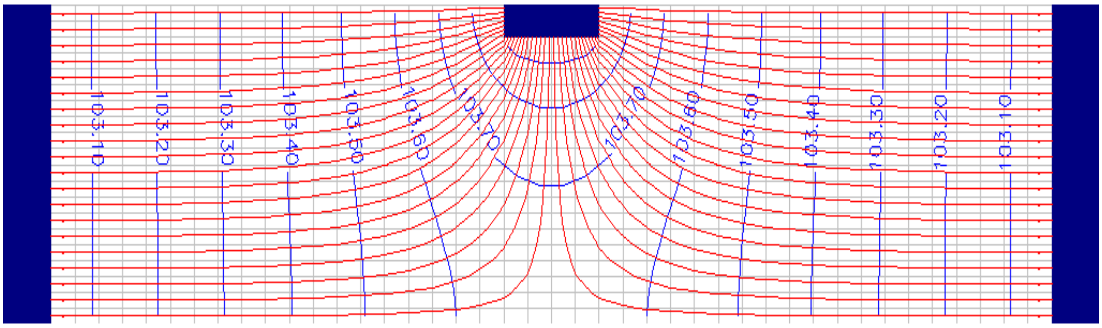

Figure 1: Flow and potential lines in a cross-section of a slightly penetrating river.

run) the trench is located in three different FD cells. Thus in that case one expects that the orientation of the trench will affect the results (details in Appendix 3 online). Indeed the total lateral (i.e. in the E-W direction) discharge is 94,838 whereas the longitudinal (i.e. N-S direction) discharge is 39,778. The ratio of seepage to the NS versus EW is now: 0.42.

Third case (third run). (See Figure 5): The parameters are essentially the same as for Case 2, except that the clogging layer thickness is 50 and its conductivity is 100. The conductivity of the clogging layer is the same as that of the aquifer (which is isotropic). There are many more cells (10) with a trench segment in it.

The size of the square cells is 205. This is also the length of each trench segment within a cell. That size was chosen so that the center of each adjacent cell is located precisely at a distance which is twice the aquifer thickness from the bank of the trench. With that distance it is particularly easy to compare with the flow calculated using the analytical SAFE approach. The MODLFLOW calculated total lateral discharge E-W was 67,860 whereas the total discharge in the direction NS is 9,798.4. The ratio is: 0.1444. For one segment the ratio was 1.0, for 3 segments it was: 0.42. The conclusion of these 3 tests is that for MODFLOW to apportion properly the seepage laterally or longitudinally the river reach must be divided into multiple segments.

Fourth case (fourth run; Figure 5): The purpose of this case is to compare the MODFLOW results and the results using the SAFE approach. The parameters are precisely the same as for Case 3 except that the aquifer geologic layer is divided into 11 calculation sublayers. The boundary condition at the river is now one of a prescribed head, 120. There is no clogging layer.

The **MODFLOW** calculated flow from cell (7, 3) to cell (7, 2), in other words EW flow to the left, is 33,151. That flow can be evaluated also using the SAFE approach. The SAFE dimensionless conductance can be calculated as a function of the normalized wetted perimeter, $W_p^N = \frac{2B+2H}{D}$, i.e. the wetted perimeter of the cross-section of the river, W_p, divided by the aquifer thickness, D, where H is depth, and B is half surface width and of the degree of penetration, $d_p = \frac{H}{D}$ (Morel-Seytoux 2013). The value of r can be determined from the knowledge of the zero degree of penetration (flat) case which is: $\Gamma_{flat} = \frac{1}{2\{1+\frac{1}{\pi}\ln(\frac{2}{1-\kappa})\}}$ (12a) where $k = e^{-\frac{pW_p^N}{2}}$ (12b)

and the relation $\Gamma = \Gamma_{flat}\{1+a_1 d_p + a_2 (d_p)^2\}$ (13)

The parameters a_1 and a_2 appearing in Equation.(13) are listed in Table 1. Using Equation.(13) the value of for a non-penetrating case

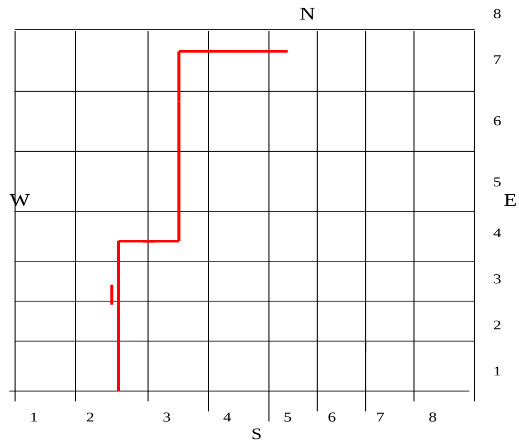

Figure 2: The blue line is the actual river channel and the red segments represent a possible stream schematic representation in the Finite Difference (FD) grid network.

with width 10 and an aquifer depth of 100 is 0.273. Using Equation.(1) the discharge is: 100x205x0.273x(120 – 116.24) = 21.042, 100 being the aquifer conductivity, 205 the river segment length, 120 the river head and 116.24 the average head calculated by MODFLOW in cell (7,2) between layers 1, 6 and 11, the top, middle and bottom layers, which were respectively 116.25, 116.24 and 116.24. Note that at the far distance from the river bank the head is essentially constant in the vertical direction, which verifies that at the distance twice the aquifer thickness from the river bank the flow is horizontal. The calculated SAFE value is much lower than the MODFLOW calculated one. The relative difference is 36%.

The reason for this large difference is that the width of the cell that contains the river is much wider than the river. As a result the river head boundary condition, instead of being applied uniquely at the river, in this use of MODFLOW, was applied to the entire width of the layer 1, the top layer. It is tantamount to make the river width to be the cell width, which is 205, thus making B looks as if it had the value 102.5. In addition the trench head boundary condition is not applied at the top of sublayer 1 but is applied to the entire sublayer 1. It is as if the trench penetrated a depth of 100/11 = 9.09 that is 1/11 of the aquifer thickness. It is as if the wetted perimeter was 205 + 2x9.09 = 223.2 and the degree of penetration was 0.091, which is almost 10%. The value of r_{flat} for this situation is 0.406. The associated value of r is: 0.406[1 + 0.919/11 - 1.34/ (11)2] = 0.432. Using Equation. (1) the

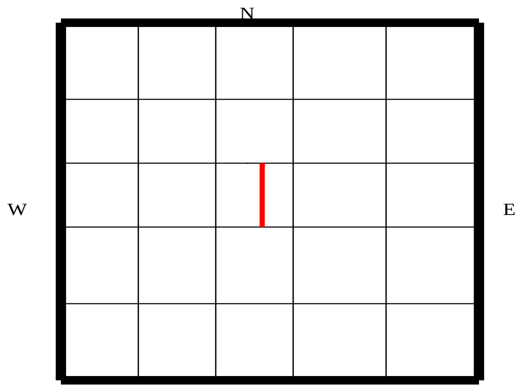

Figure 3: Trench located entirely within a grid cell.

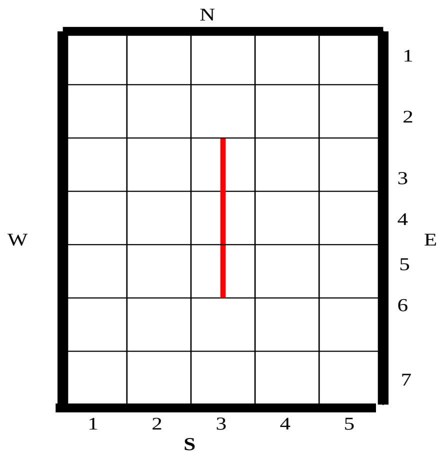

Figure 4: Second case (run). The trench is now divided into 3 different FD cells.

discharge is thus Q = 100x205x0.432x(120 – 116.24) = 33,298 to be compared with the numerical value of 33,151 for a difference of 0.4 %.

This 4th run illustrates a couple of points. First, if a boundary condition of head is to be applied without the presence of a clogging layer the width of the cell that contains the river should be the same as that of the river (or smaller) and the thickness of the first sublayer should be very small compared to the other sublayers because otherwise it makes the river in fact penetrate the aquifer to the full thickness of the top sublayer. Second under the assumption of this numerical MODFLOW run the SAFE approach predicts the same result pointing out its advantage since it obtains the correct result without having to make runs with 11 sublayers. We presume that users of MODFLOW would not insert the river within a much larger cell if they were to apply a prescribed river head without including a clogging layer. If they used a much larger cell then they would have to introduce an artificial clogging layer resistance to compensate for the conceptual error. Based on this information we can proceed with the presence of sinuosity in the river.

Case of a Sinuous River. Fifth Case (Fifth Run; Figure 6)

To finally make a comparison to verify the correctness of the SAFE

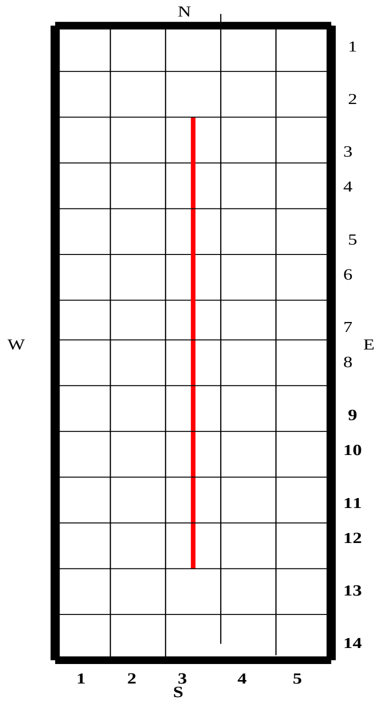

Figure 5: For runs 3 and 4.

approach, including curving of the river, as just illustrated for run 4, one needed to have a much more refined grid in both the horizontal and the vertical planes. In this situation there will be *asymmetry of heads on the two sides* of the river.

Figure 6 illustrates that geometry. Cells with light green are a third-type boundary (**MODFLOW**'s general-head boundary, **GHB**). Dark blue is a constant-head boundary (which in this model is equivalent to a river boundary with no clogging layer). The parameters for this run are as follows. The river width is 50. Each grid in the horizontal plane is square of size 10. The aquifer thickness of 30 is divided into 11 computing sublayers. The top sublayer has a small thickness of only 0.5 while the thickness of the 10 others are 2.95; (0.5 + 2.95x10 = 30). That should represent closely a case of no penetration of the river.

Figure 7 shows the contour lines of heads in the top sublayer.

Analysis for column 34: At first we shall look at column 34 to find the seepage rate out of cell (7, 34; little green square in Figure 7) toward the North. For this river width of 50, no penetration and an aquifer thickness of 30, the value of the one-sided is 0.402. The river head is 120 and the vertically average head in cell (1,34) for sublayers 1, 6 and 11, the top, middle and bottom sublayers, located at the far distance, is (110.282 + 110.388 + 110.359)/3 = 110.343. The head difference is (120 – 110.343) = 9.657. The discharge using the SAFE dimensionless conductance of value 0.402 is thus: $Q_{safe\ N}$ = 9.657*0.402*K*L = **3.882KL**, (K=100 and L = 10).

For the evaluation of the numerical discharge the differences in heads between row 2 and row 1 in layers 1, 6 and 11 are respectively: 111.720 – 110.282 = 1.438; 111.733 – 110.388 = 1.345 and 111.584 – 110.359 = 1.225. On the average it is 1.336. Applying Darcy's law for the entire 11 layers then: = 1.336*30 (the aquifer thickness)/10 (the

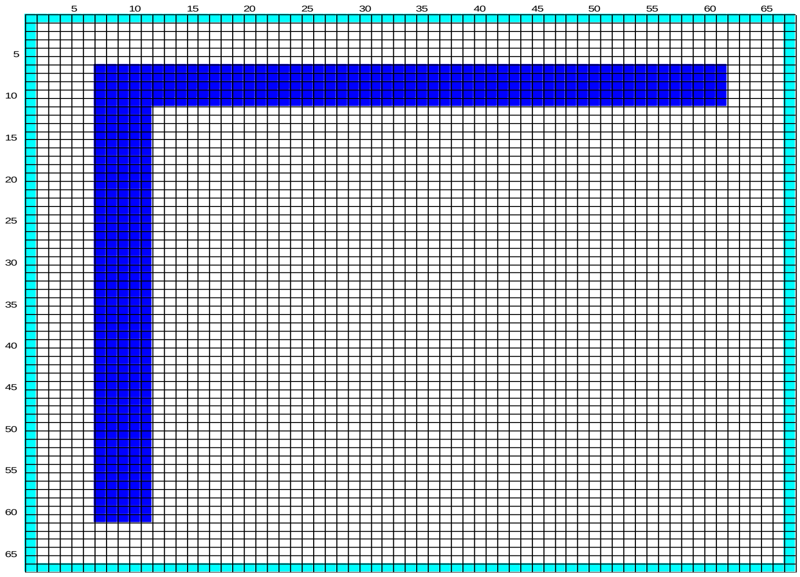

Figure 6: Curving river with multiple grids below the bottom of the river.

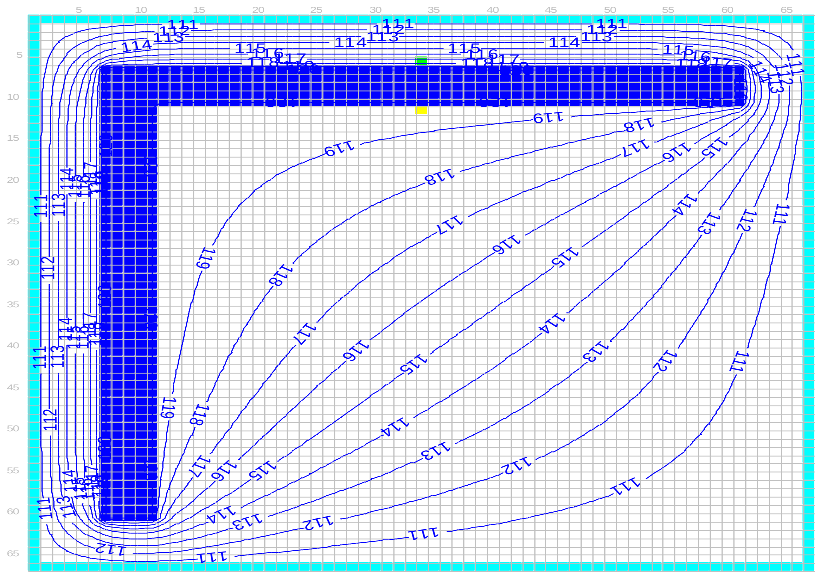

Figure 7: Contour lines of heads in the top sublayer.

cell horizontal size)*K *L= 4.008KL. Next we look at row 11 for the same column 34 to find the total seepage rate out of cell (11, 34; little yellow square in Figure 7) toward the South.

The average head in row 17 (at the far distance) in column 34 in sublayers 1, 6 and 11 is: (118.592 + 118.592 + 119.554)/3 = 118.913. The head difference is 120 - 118.913 = 1.087. The discharge using the SAFE dimensionless conductance of value 0.402 is thus: Q_{safeS} = 1.087*0.402*KL = **0.437KL**.

For the numerical value the differences in heads between row 16 and row 17 in layers 1, 6 and 11 are on the average 0.109. Applying Darcy's law for the entire 11 layers then:

Q_{numS} = 0.109*30 (the aquifer thickness)/10 (the cell size)*KL = **0.327KL**

Note that the dimensionless conductance used so far is the one for the case of symmetry of heads, which is not at all the case for this run, as clearly shown on Figure 7. The dimensionless conductance needs a correction. However without any correction [2,3] the sum of the discharges on both sides of the river thus calculated provide the correct answer. Checking:

$$Q_{safeN\,\&\,S} = Q_{safeN} + Q_{safeS} = 3.882KL + 0.437KL = 4.319KL$$

and for the numerical value:

$$Q_{numN\,\&\,S} = Q_{numN} + Q_{numS} = 4.008KL + 0.327KL = 4.335KL$$

The relative difference of values is 0.4 %. One can conclude that

Table 1: Values of coefficients in Equation.(13) for curve fitted values of analytical Γ.

Range of W_p^N	Range of d_p	a_1	a_2
$W_p^N \le 1.0$	$d_p \le 0.2$	0.890	-2.430
$W_p^N \le 1.0$	$0.2 \le d_p \le 0.5$	0.538	-0.387
$1.0 \le W_p^N \le 3.0$	$d_p \le 0.2$	0.819	-1.340
$1.0 \le W_p^N \le 3.0$	$0.2 \le d_p \le 0.5$	0.672	-0.542

the two approaches give the same answer.

Analysis for column 21: The analysis is repeated for column 21 to find the seepage rate out of cell (7, 21) toward the North. For this river width of 50, no penetration and an aquifer thickness of 30, the value of the one-sided is 0.402. The river head is 120 and the vertically average head in cell (1,21) for sublayers 1, 6 and 11, the top, middle and bottom sublayers, located at the far distance, is (110.282 + 110.388 + 110.395)/3 = 110.355. The head difference is (120 – 110.355) = 9.645. The discharge using the SAFE dimensionless conductance of value 0.402 is thus: Q_{safeN} = 9.645*0.402*K*L = **3.877KL**, (K=100 and L = 10).

For the evaluation of the numerical discharge the differences in heads between row 2 and row 1 in layers 1, 6 and 11 are respectively: 111.720 – 110.282 = 1.438; 111.733 – 110.388 = 1.345 and 111.735 – 110.395 = 1.34. On the average it is 1.374. Applying Darcy's law for the entire 11 layers

then Q_{numN} = 1.374*30 (the aquifer thickness)/10 (the cell horizontal size)*K *L= **4.123KL**. Next we repeat the analysis for the same column 21 to find the total seepage rate out of cell (11, 21) toward the South.

The average head in row 17 (at the far distance) in sublayers 1, 6 and 11 is: (119.233 + 119.232 + 119.232)/3 = 119.232. The head difference is 120 – 119.232 = 0.768 The discharge using the SAFE dimensionless conductance of value 0.402 is thus: Q_{safeS} = 0.768*0.402*KL = **0.309 KL.** For the numerical value the differences in heads between row 16 and row 17 in layers 1, 6 and 11 are (119.307 – 119.233) = 0.074, (119.306 – 119.232) = 0.074, (119.304 – 119.232) = 0.072 on the average 0.073. Applying Darcy's law for the entire 11 layers then: = 0.073*30 (the aquifer thickness)/10 (the cell size)*KL = 0.219KL Note again that the dimensionless conductance used so far is the one for the case of symmetry of heads, which is not at all the case for this run, as clearly shown on Figure 7. However without any correction [2,3] the sum of the discharges on both sides of the river thus calculated provide the correct answer. Checking:

$$Q_{safeN\&S} = Q_{safeN} + Q_{safeS} = 3.877KL + 0.309KL = 4.186KL$$

and for the numerical value:

$$Q_{numN\&S} = Q_{numN} + Q_{numS} = 4.123KL + 0.219KL = 4.342KL$$

The relative difference of values is again 0.4 %. One can conclude that the two approaches give the same answer. The SAFE approach has the advantage of necessitating a minimum of numerical work as there is no need to divide the aquifer in a number of sublayers and no need to use a horizontal grid size smaller than the river width.

Conclusion

The calculations discussed for columns 34 and 21 were repeated for a number of other columns with essentially the same match between the SAFE approach and the use of the very fine discretized MOFLOW code. We can conclude that the SAFE approach is applicable for rivers that deviate from a straight path.

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