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Research Article

Fractional Calculus Approach to the Deformation Field near the Fault Zone

Yamasaki K[1]* and Yajima T[2]
[1]Department of Planetary Sciences, Kobe University, Japan
[2]Mechanical Systems Engineering, Utsunomiya University, Japan

***Corresponding author:** Kazuhito Yamasaki, Department of Planetary Sciences, Faculty of Science, Kobe University, Kobe, Japan

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Abstract

Based on fractional calculus, we considered the fault zone as a typical example of the deformed medium including internal structures characterized by the fractional dimension. The fractional notation of the Green's function for the generalized Laplace field gives a relationship between the fractional dimension and the fractional order of the equation. We applied the relationship to the displacement field around the strike-slip fault characterized by the generalized Laplace equation. From the observed data for the strike-slip fault, we calculated the fractional dimension of the fractional deformation field. This result implies that the non-locality of the deformation field increases toward the fault. Moreover, we suggest the heuristic method of deriving the displacement field of the fractional Navier equation in the semi-fractional case.

Keywords: Fault zone; Fractal; Fractional calculus; Displacement field; Deformation

Introduction

There has been ongoing theoretical interest in deformed mediums with internal structures including discontinuities [1-5]. In particular, internal structures in the lithosphere influence various deformation phenomena, such as complex fracturing in a fault zone [6-10]. From a geometrical viewpoint, the concept of the fractal dimension is useful for characterizing the complexity of such internal structures [11-14].

As pointed out by Tarasov [15], most of the processes associated with complex systems including fractal media have nonlocal dynamics in time and space. On the other hand, the fractional calculus is a powerful tool for describing physical systems that have long-term memory and long-range spatial interactions. Therefore, close connections exist between fractional calculus and the dynamics of many complex systems, including fractal media [15,16]. This paper considers deformation fields in a fault zone from the viewpoint of fractional analysis.

To quantitatively discuss the coupling between a fault zone and deformation fields, a study of Green's function will be quite helpful because this function includes fundamental properties of the deformation field generated by the corresponding source [17]. Thus, this paper applies the fractional Green's function to the deformation fields in a fault zone. In particular, we focus on the fractional Laplace field for the following reasons.

Previous studies [9,10,18] have shown that the Laplace equation is significant for describing scale-invariant properties of fracturing in a lithospheric plate with internal discontinuities. Generally, the Laplace equation has been used to describe diffusion-limited aggregation such as complex fracturing characterized by the fractal dimension (fractional dimension) [19]. The Laplace equation with both homogeneous elastic coefficients [20,21] and non-homogeneous elastic coefficients [22] has been analyzed in a complex deformation field. This paper considers the fractional property of the derivative itself to analyze deformed mediums, including internal structures such as a fault zone. For this purpose, the paper considers the Green's function for the Laplace equation with the fractional derivative

$$(-\Delta)^{a/2} f=0 \qquad (1)$$

where a is a fractional value and f is a function. When $a=2$, this equation becomes the standard Laplace equation.

In the analysis of a fractional derivative such as Equation (1), we should use fractional calculus [23-25]. Caputo and colleagues have done pioneering work on fractional analysis in various subjects, including the solid earth science and the biosciences [26-30]. For instance, they applied the fractional derivative to viscoelastic models and enabled the description of power-law relaxation and the memory effect [31].

Caputo [32] introduced the fractional derivative that is often used in the fractional calculus as well as the Riemann-Liouville fractional derivative. From a differential geometric standpoint, fractional calculus has been developed based on the Caputo fractional derivative [33-35] because of zero values for actions on constants. On the other hand, fractional Fourier transforms have also been used particularly for the fractional Green's function [16] and have been applied to various phenomena, such as general turbulence systems and incompressible two-dimensional flows in geophysical fluid dynamics [17,36]. Because we want to apply the fractional Green's function of Equation (1), this paper uses fractional Fourier transforms.

Methods

Review of the fractional derivative

The fractional derivative is defined via a fractional integral, such as the Riemann-Liouville integral [23-25]. We apply the Fourier integral because this approach is useful for deriving a Green's function. This paper defines the Fourier transform and inverses as follows:

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$$\hat{A}(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dxA(x)e^{ikx}, \quad (2)$$

$$A(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dk\hat{A}(k)e^{-ikx}. \quad (3)$$

The derivative of the function results in:

$$\frac{d^n}{dx^n}A(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dk(-ik)^n\hat{A}(k)e^{-ikx}, \quad (4)$$

where n is an integer. Replacing n with a positive real number a defines the fractional derivative [24]:

$$\frac{d^a}{dx^a}A(x)=D_x^aA(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dk(-ik)^a\hat{A}(k)e^{-ikx}. \quad (5)$$

For instance, consider the function $A(x) = |x|^{-k}$ $(x<0)$. From Equations (2) and (5), we obtain:

$$\frac{d^a}{dx^a}|x|^{-k}=D_x^a|x|^{-k}=\frac{\Gamma(k+a)}{\Gamma(k)}|x|^{-k-a}, \quad (6)$$

where Γ is the Gamma function defined by:

$$\Gamma[k]=\int_0^{\infty}dtt^{k-1}e^{-t}. \quad (7)$$

In the next section, we review the Green's function of the fractional Laplacian based on the fractional derivative.

Fractional order of derivatives and dimension

Let us consider the following fractional Laplacian of any order

$$(-\Delta)^{a/2}f(r) = -q(r), \quad (8)$$

where $f(r)$ and $q(r)$ are the function of the position r. For instance, in some geophysical fluids, $f(r)$ is the stream function, $q(r)$ is the scalar field advocated by the velocity field, and the parameter a characterizes the property of a generalized turbulence system [17]. The Green's function for the fractional Laplacian of any order a is already known mathematically [17,37,38]. The fractional Laplacian in Equation (8) is also applied to the fractional diffusion equation and the fractional wave equation [38,39].

From the Fourier transform of (8) with the point source $-\delta(r)$, we obtain a Green's function in Fourier space:

$$\hat{G}(k)=-\frac{1}{2\pi}\frac{1}{|k|^a}. \quad (9)$$

Thus, a Green's function in the physical space is given by the inverse Fourier transform of (9) [17]:

$$G(r)=-\frac{1}{2^a\pi}\frac{\Gamma[(2-a)/2]}{\Gamma[a/2]}r^{a-2}, \quad (10)$$

where we consider the case $1\le a\le 3$ and $a\ne 2$. For the general case including $a<0$ and applications, see [17,36].

Equation (10) shows that the distance dependence of a Green's function in terms of the fractional order of derivatives is given by:

$$G(r) \propto r^{a-2}. \quad (11)$$

For instance, we have $G\propto 1/r$ for $a=1$. This is in agreement with the distance dependence of a Green's function in three-dimensional space. Moreover, for $a=3$, we have $G\propto r$, which is in agreement with the one-dimensional case. These results imply the following relationship between the fractional orders of derivative a and the dimension D:

$$a + D=4. \quad (12)$$

In fact, the distance dependence of a Green's function in terms of the dimension D is given by:

$$G(r) \propto r^{2-D}. \quad (13)$$

By comparing Equation (11) and Equation (13), the relation of (12) is also supported. Because a is a real number, (12) means that the dimension D is not necessarily an integer, such as the case of a fractal dimension. Thus, in this paper, we call D the fractional dimension.

Note that we should distinguish the fractional dimension from the space dimension. For instance, even if a phenomenon in two-dimensional space can be characterized by the fractional dimension, the dimension of space, in which the phenomenon is embedded, remains two. This is similar to the case of fractal dimension. For instance, we consider coastlines in two-dimensional space. The dimension of space remains two even if the complexity of the coastline's measured length is characterized by the fractal dimension.

Results

Dislocation model of strike-slip fault

From the viewpoint of the Volterra dislocation, the screw dislocation corresponds geometrically to the strike-slip fault [40-42]. In three-dimensional space(x_1,x_2,x_3), let us consider the screw dislocation (strike-slip fault) with the dislocation line along the x_3-direction. The boundary condition at the dislocation is that the displacement discontinuity across the dislocation surface is equal to the fault slip. Adopting a radial coordinate system (r,θ) centered on the dislocation line (the x_3 axis), the slip fault is related to the displacement field u_3 (Figure 2 in [43]). In this case, Hooke's law (i.e., the elastic constitutive law) shows that there are only two nonzero stresses: $\sigma_{13}=\mu\partial_1 u_3$ and $\sigma_{23}=\mu\partial_2 u_3$, where μ is the Lame constant. On the other hand, the equilibrium equation gives $\partial_1\sigma_{31}+\partial_2\sigma_{32} =-F$, where F is the external force. Then, the displacement field $u_3=U(x_1,x_2)$ satisfies

$$\partial_1\partial_1 u_3+\partial_2\partial_2 u_3 =\Delta U=-F. \quad (14)$$

To consider the fractional effect of internal structures on the deformation field in the fault zone, we extend this equation such as in (8):

$$(-\Delta)^{a/2} U= -F. \quad (15)$$

Then, we consider that the point force (double-couples type; [43]), Equation (13) gives the displacement field in terms of the fractional dimension D:

$$U=Cr^{2-D}, \quad (16)$$

where C is a constant. From section 3, the relationship between a in Equation (15) and D in Equation (16) is given by the following:

$$a + D =4. \quad (17)$$

Equations (16&17) show that the displacement in fractional space can be characterized by the fractional derivative.

For instance, we can consider the displacements that occurred during the 1992 magnitude -7.3 Landers earthquake (Figure 1; data from [44]). Based on the observed data in Figure 1, we estimate the relative variation in the fractional dimension D of Equation (16). We set the value of D to 1.00 at the farthest point $r\approx 27$. In this case, Figure 2 shows that the fractional dimension decreases with proximity to the

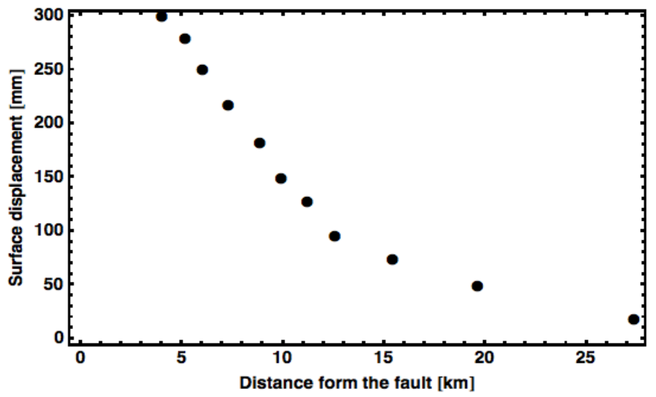

Figure 1: Surface displacement during the 1992 magnitude 7.3 Landers earthquake plotted against distance from the fault (data from [44]).

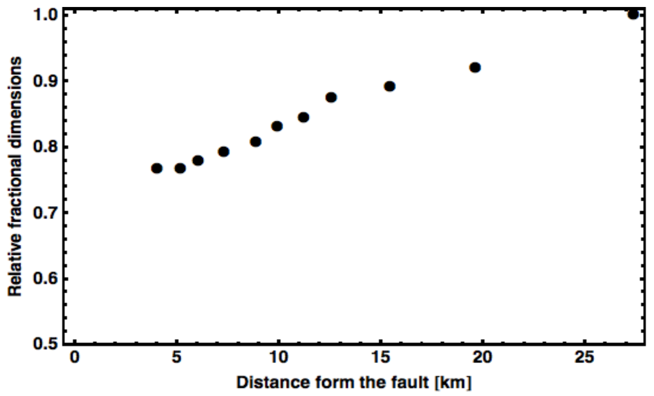

Figure 2: Fractional dimensions estimated from the data in Figure 1.

fault. In section 5, we discuss the meaning of this variation.

Fractional navier equation

In solid earth science, the Navier (Cauchy) equation is often used, as is the Laplace equation; thus, we consider the fractional Green's function for the Navier equation. First, we review the integer case of a Green's function, and then we extend this derivation process to the fractional case.

The Navier equation is derived from the combination of three basic equations, i.e., the equation of motion ($\rho d^2u_i /dt^2 = \partial_i\sigma_{ji}+F_i$), the strain-displacement equation ($\varepsilon_{ij} = (1/2)(\partial_i u_j + \partial_j u_i)$), and the constitutive equation (Hooke's law; $\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}$), where u_i is the displacement field, ε_{ij} is the strain field, σ_{ij} is the stress field, μ and λ are the Lame constant, F_i is an external force, and ρ is the density of mass. Under the condition of equilibrium, the Navier equation in terms of the displacement field is given by the following:

$$\mu\partial_l\partial_l u_i + (\lambda+\mu)\partial_i\partial_k u_k = -F_i. \qquad (18)$$

From the point force acting at the point ξ, we obtain the equation in terms of a Green's function g_{ij}:

$$\mu\partial_l\partial_l g_{ij} + (+\mu)\partial_i\partial_k u_{kj} = -\propto_{ij}\beta(x_i-\xi_i), \qquad (19)$$

where $\delta(x_i - \xi_i)$ is a triple Dirac delta. In this case, we consider the following form of the solution:

$$g_{ij} = \alpha\frac{\delta_{ij}}{R} + \beta\frac{r_i r_j}{R^3}, \qquad (20)$$

where $r_i = x_i - \xi_i$ and $R = |r| = \sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2}$. Forms such as (20) are not general forms, but are often used in solid-earth science to consider the deformation field around the dislocation (fault) (e.g., [43]). The parameters α and β are the constants determined by substituting (20) into equation (19) as follows:

$$-2\beta + \frac{\alpha-\beta}{1-2v} = 0, \qquad (21)$$

$$-16\beta\pi(1-v) + \frac{1}{\mu} = 0, \qquad (22)$$

where v is a Poisson's ratio. From (21) and (22), the concrete form of (20) is given by:

$$g_{ij} = \frac{1}{16\pi\mu(1-v)}\left[(3-4\mu)\frac{\delta_{ij}}{R} + \frac{r_i r_j}{R^3}\right]. \qquad (23)$$

This is the well-known form of the Green's function for the Navier equation [45]. Next, we extend the above derivation process to the fractional case.

We replace the ordinary derivative operator with the fractional derivative operator D_i^a. In this case, the basic equations used to derive Equation (18) are rewritten as $\rho D_t^{2a} u_i = D_i^a\sigma_{ji} + F_i$ and $\varepsilon_{ij} = (1/2)(D^a_i u_j + D^a_j u_i)$. Regarding the constitutive equation, we can use the ordinary one because it does not include the derivative term. In this case, Equations (18&19) are rewritten as follows:

$$\mu(D^a_l)^2 u_i + (\lambda+\mu)D^a_i D^a_k u_k = -F_i, \qquad (24)$$

$$\mu(D^a_l)^2 g_{ij}^d + (\lambda+\mu)D^a_i D^a_k g_{kj}^d = -\delta_{ij}\delta(x_i-\xi_i). \qquad (25)$$

Moreover, from Equation (20), we assume the Green's function to be in the following form:

$$g_{ij}^d = A\frac{\delta_{ij}}{R^{d-2}} + B\frac{r_i r_j}{R^d}, \qquad (26)$$

where $d(\neq 2)$ is a positive real number and A and B are coefficients. Just as the coefficients of Equation (20) are determined by Equation (19), the coefficients A and B of Equation (26) are determined by Equation (25).

To determine A and B, we consider the semi-fractional dimension; i.e., the integer dimension with a small perturbation (real number), in the following analysis. Moreover, to simplify the calculations, we introduce the concept of transformation between the ordinary derivative and the fractional derivative. For instance, the fractional derivative of the power law function x^{-d} is given by the following formal form:

$$D^a_x x^{-d} = E(d,a)x^{-d-a}, \qquad (27)$$

where $E(d,a)$ is a function of d and a. As shown in section 2, if $d=k$, we have $E(k,a) = \Gamma[k+a]/\Gamma[k]$. On the other hand, the ordinary derivative gives $\partial_x x^{-d} = -dx^{-d-1}$. Therefore, we obtain the following transformation:

$$D^a_x x^{-d} = F(d,a,x)\partial_x x^{-d}, \qquad (28)$$

where $F(d,a,x) = -E(d,a)x^{-a+1}/d$. Equation (28) means that the fractional derivative of the power law function is equal to the product of the corresponding function and the ordinary derivative. Based on this result, we give the heuristic method of solving problems, i.e., assume the following transformations:

$$(D^a_l)^2\partial_j\partial_i R^{4-d} = G(4-d,a,x_l)\partial_i\partial_j\nabla^2 R^{4-d}, \qquad (29)$$

$$D^a_i D^a_j R^{2-d} = H(2-d,a,x_i,x_j)\partial_i\partial_j R^{2-d}, \qquad (30)$$

$$D^a_i D^a_k \partial_k \partial_j R^{4-d} = I(4-d,a,x_i,x_k)\partial_i \partial_j \nabla^2 R^{4-d}, \tag{31}$$

$$(D^a_l)^2 R^{2-d} = J(2-d,a,x_l)\nabla^2 R^{2-d}. \tag{32}$$

Under relations (29)–(32), we can determine the coefficients A and B by substitution of Equation (26) into Equation (25):

$$A = -\frac{B}{(d-2)H}\left[(d-5)(1-2v)G + (d-5)I + H\right], \tag{33}$$

$$B = \frac{1}{J}\left[-\frac{(d-5)(1-2v)}{d-2}\frac{G}{H} - \frac{d-5}{d-2}\frac{I}{H}\right]^{-1}\left[-\frac{\Gamma[d/2]}{\mu(2-d)2\pi^{d/2}}\right]. \tag{34}$$

Therefore, Equation (26) gives the fractional Green's function:

$$g^d_{ij} = -\frac{1}{J}\frac{\Gamma[d/2]}{\mu(2-d)2\pi^{d/2}}\left[-\frac{(d-5)(1-2v)}{d-2}\frac{G}{H} - \frac{d-5}{d-2}\frac{I}{H}\right]^{-1} \times\left[\frac{r_i r_j}{R^d} - \frac{1}{(d-2)H}\left\{(d-5)(1-2v)G + (d-5)I + H\right\}\frac{r_{ij}}{R^{d-2}}\right]. \tag{35}$$

When d=3 and the fractional derivative becomes the ordinary one (i.e., G=H=I=J=1), this relation is in agreement with the ordinary Green's function (23).

Discussion

The Laplace equation has been used to describe complex phenomena characterized by the fractal dimension, such as diffusion-limited aggregation, dielectric breakdown, and fracturing [19]. In particular, Nagahama and Teisseyre [9] used the micromorphic continuum to derive the generalized Laplace equation (the local diffusion-like conservation equation) for strains and clarified the scale-invariant properties; i.e., fractal properties of fracturing in a lithospheric plate with microstructure under steady non-equilibrium strain flux through plate boundaries.

This paper generalizes the Laplace equation in the sense of the fractional derivative, Equation (15), and uses the relationship between the fractional dimension D and the fractional derivative order a, such as Equation (17): $a + D = 4$. Because a is the derivative order, larger a is related to more non-local phenomena. From $a + D = 4$, smaller D is also related to more non-local phenomena. Therefore, we can estimate the degree of non-locality by measuring the fractional dimension D. Figure 2 shows that the fractional dimension decreases toward the fault. This implies that the non-locality of the deformation field increases toward the fault.

On the other hand, previous fractal analysis of fractured rocks showed that the fractal dimension increases toward the fault [46,47]. The fractal dimension of fracture patterns depends on the energy density for fracturing [48], and the fractal dimension can be employed as an independent measure of the energy distribution near faults [47]. Therefore, if the relationship between the fractional dimension of this study and the fractal dimension of previous studies can be derived, the non-locality estimated by the fractional dimension may be related to the energy distribution estimated by the fractal dimension. This is a topic for future research.

We generalized the Navier equation in the sense of the fractional derivative: Equation (24). We suggested the heuristic method to derive the Green's function, Equation (35), by the point force, and

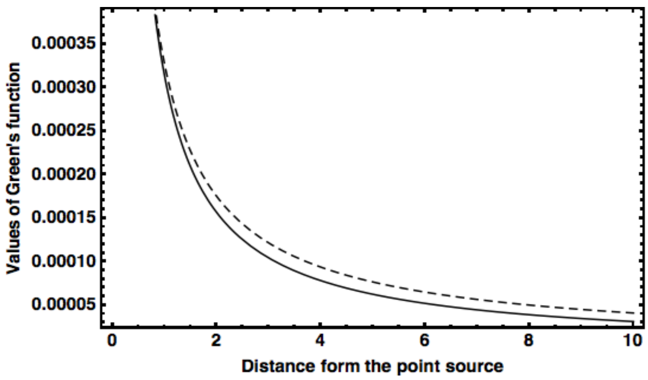

Figure 3: Green's function plotted against the point source. The solid and broken lines give the integer case d=3 and the sub-fractional case d=2.9, respectively.

it is in agreement with the ordinary Green's function (23) when d=3 and the fractional derivative becomes the ordinary one. To clarify the characteristic of the fractional of a Green's function, let us calculate the value of g^d_{12} for the integer case d=3 and the semi-fractional case d=2.9 (i.e., G≈H≈I≈J≈1). Figure 3 shows a plot of the value of g^d_{12} against the distance from the point source calculated at μ=30 [GPa] and v=0.25. The solid line indicates d=3 and the broken line indicates d=2.9. It can be seen that the difference between the value of the fractional case and the value of the integer case increases with distance from the point source. That is, the fractional Green's function shows long-range correlation compared with the standard Green's function.

Conclusion

(1) The fractional Laplace equation shows that the fractional dimension defined in this paper is related to the fractional order of the equation, as described by Equation (17). Equation (17) indicates that the fractional dimension is inversely proportional to the non-locality related to the fractional order of the equation. From this relation and the observed data, it was found that the fractional dimension decreases; i.e., the non-locality of the deformation field increases, toward the fault.

(2) Because the fractal dimension increases toward the fault, it is implied that the non-locality estimated by the fractional dimension is related to the energy distribution estimated by the fractal dimension.

(3) We suggested and computed the component of the Green's function for the fractional Navier equation in the semi-fractional dimension case, and it was found that the fractional Green's function shows a slow decreasing of the displacement compared with the standard Green's function.

For the conclusion will be more convincible, we plan in the near future to apply our method to other cases in which the precise data for displacement field were obtained just like the 1992 Landers earthquake.

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