Review Article

How the Cost of Control Affects the Cost of Shares of Various Stakes

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Abstract

The main purpose of this paper is to create a mathematical apparatus for calculating premiums for control and discounts for the lack of control when calculating share interests in business. The paper covers the issue of estimating premium and discount rates for the presence or lack of control when switching from one to another level of value taking into account changes in a controlling number of shares exercised by the owners of corresponding stock shares, equity holding structure, market conditions and applicable laws covering stock corporations. The paper also provides corresponding mathematical tools and calculation examples.

Keywords: Control Premium; Discount for the Lack of Control; A Stake; Block/Parcel of Shares; Amount of Control

Introduction

Valuators, investment bankers and financial advisors practically often deal with bidding which is aimed to identify the value of the shares when acquiring strategic blocks. The pre-requisite for a successful deal is to provide a large block holder with a premium to the current stock exchange value taking into account that the owners of such stocks hold more extensive powers if compared to the powers of minority shareholders. At the same time, it is necessary to remember that such stocks are usually less liquid if compared to book shares because they are more expensive, require more time for the search of buyers and often need agreement with other shareholders, creditors and regulatory authorities.

The nature of corporate control in terms of pricing is determined by the following: for the buyer or the owner of the controlling strategically valuable (holding powers of control) block of shares – (1) by the power of dominating disposal of the largest part of shared proprietary interests of the shareholders involving potential possibility of rearrangement of their shared interests which were initially rated on an equitable basis [1]; (2) by the possibility of combining the controlled company's resources with other assets of the buyer/owner (synergy effect) for the seller (of the strategically valuable shares of stock exercising powers of control) – by an additional incentive for the sale of their shares which is provided by the premium to the market value of these shares (premium for quick buying). When there is no such premium, the majority of large shareholders would not like to sell their shares [2].

The power of dominating disposal by the major part of the shareholders' property is determined by legal and normative delegation of a range of powers to manage property and financial and operational operations of the companies by their manager appointed usually by the largest shareholders of the company. Delegation of powers means voluntary (for some minority shareholders it is forcibly volunteered) waiver of the shareholders' property rights to dispose of their own property. The managers appointed by the majority of

the shareholders, acting in the interests of the company entrusted to them, first of all serve the interests of the largest shareholders who have chosen them. For example, sometimes the managers appointed by the largest shareholders divert some part of the company profit to their other captive companies using current law imperfections thus infringing the interests of the other shareholders. They use widely known schemes like transfer pricing, asset stripping, and allocation of profit and cost centers within specially created holding structures. All this can cause difference between actual participation interest in the profit of different groups of shareholders and their par value interest. The degree of such difference varies from company to company depending on the robustness of the shareholders' interests and on the presence of absence of quasi-legal schemes diverting profit from the company. Minimum degree of this disproportion is commonly found in transparent companies having a balanced shareholders' structure, maximum degree is in nontransparent companies who often violate the rights of minority shareholders. The disproportion may result into differences in the value of the shares from different parcels. Usually, the value of one share in strategically valuable large voting parcels (absolutely controlling (75%), controlling (50% +1 share) and/or blocking (25% + 1 share) parcels) is higher than the value of a share from minority parcels. This difference is larger when the interests of minority shareholders are violated, for example, when the structure of the capital stock is less favorable for them with the absolutely controlling parcel of shares belonging to one person, or when top-managers of the company play with dividends and additional emission of shares by private subscription resulting into diversion of the socks of minority shareholders.

Reallocation of property rights described above is the source of a special estimation subject – control – which has its own value like indispensable from the company goodwill. Generally speaking the value of control rests on two components: reallocation of rewards from minority shareholders to controlling shareholders ("grabbing of the biggest piece of pie" factor) and possibility of combination of the controlled company resources with the resources of other companies.

Table 1: Levels of value of one share in different parcels.

Agreed notation for the calue of one share	Description of the value level
p	- value of one share from a minority parcel (constituting less than 10% of the company capital share)
$p(n_v)$	- value of one share from a strategically valuable parcel constituting 10%-25% minus one share.
$p(n_b)$	- value of one share from a blocking parcel (25%-50% minus one share)
p_{50-1}	- value of one share from a 50% parcel when there are two consolidated 50% parcels
p_{50-2}	- value of one share from a 50% parcel of shares when there is no other (except the one that is being evaluated) consolidated 50% parcel
p_{nc}	- value of one share from a controlling parcel (from 50%+1 share to 75% minus one share) that makes n_c interest in the capital stock, 50%+1 share ≤ n_c ≤ 75%-1 share
p_{sc}	- value of one share from a super controlling parcel (from 75% to 100% minus one share)
p_{100}	- value of one share from a 100% parcel of shares

The first component implies voluntary delegation of the rights of all shareholders (within operational management of their property) to a controlling group of shareholders and of the possibility of the latter to quote transfer prices for which the company makes settlements with suppliers and consumers. Delegation of the shareholders' rights to top managers appointed by controlling shareholders implies the receipt of a positive effect from their influence on quite legal decisions to fix high bonuses, salaries and other privileges as well as to pay dividends, attract of new sources of funding and perform operations with the company assets. Legal force of such decisions is secured by the decision of "their own" management on the issues not requiring the approval of the majority of shareholders and by "steamrolling" the necessary decisions at the shareholder meetings by a majority of vote. At the same time manipulations with transfer prices, such as profit withdrawal via transfers, fixing of ultra-high compensations for the top managers without preliminary approval of the rest of the shareholders, etc. are not actually lawful, and are often not lawful at all

The second component implies additional possibilities for combination of the resources of the controlled company with the resources of other companies including those affiliated to the controlling shareholders. Its characteristic feature is its legal force and the receipt of personal advantages without impairing the other shareholders of the company. For example, by virtue of the position held by a controlling shareholder of company A he/she received information he/she can successfully use for receipt of advantages in another company B belonging to him/her.

The author has created mathematical tools for quantitative evaluation of the control factor impact on the value of shares in different parcels depending on the capital stock structure and corporate management norms.

Identification and estimation of the value of control, premiums and discounts for the control

The control value can be estimated directly and indirectly. The direct method is more accurate, but the data required for it are as a rule least of all accessible (because the information on hidden income of controlling shareholders included into the costs referred to management and marketing expenses and sometimes to prime cost are not usually available for observers). By theory, estimating the value of control presents no problem if there is an adequate evaluation of the money flow generated by this control. Then estimating the value of control can be performed by discounted cash flow method1 when cost

elements are evaluated that are additional to the capitalized income flows available for the shareholders and calculated by proportion (as percentage/interest in the company capital shares).

Due to the reason that determination of the actual "control money flow" for an outsider valuator is often impossible, the control value can be estimated indirectly.

Initial conditions are simple: we can determine the value of a 100% parcel of shares by one (or several) method(s) taking into account the value of control and by methods not taking into account the value of control. Besides, stock quotes of the shares and the cost of the accepted bids for acquisition of certain shares of stock are sometimes known. If we prorate the value of a 100% parcel of shares on the basis of these values and compare it with the value obtained by estimation methods accounting for the value of control, the positive difference we receive (if any) can be regarded as the estimation of the market value of the control over the company.

Value of control is determined indirectly [2] as follows:

$$CV = MC_c - MC = 120 - 100 = 20 \tag{1}$$

$$cv = \frac{CV}{N} = \frac{20}{100} = 0,2, \tag{2}$$

$$\delta cv = \frac{CV}{MC} = \frac{cv}{p}, \tag{3}$$

where CV is full value of control over the company expressed in absolute form (money) [3];

MC is market cap without the value of control, *MC* = p. N (here and elsewhere for example *MC* = 100);

MC_c is windowed estimate of market cap with account of the value of control calculated by the methods accounting for the value of control [4], or on the basis of the cost of accepted bids for acquisition of strategically valuable shares, $MC_c = p_{100}$. *N* (here and further on elsewhere for example $MC_c = 120$);

p is stock exchange value of one share or calculated value of one share obtained by one of the methods for estimation of share values without the value of control 5, *p* = 1;

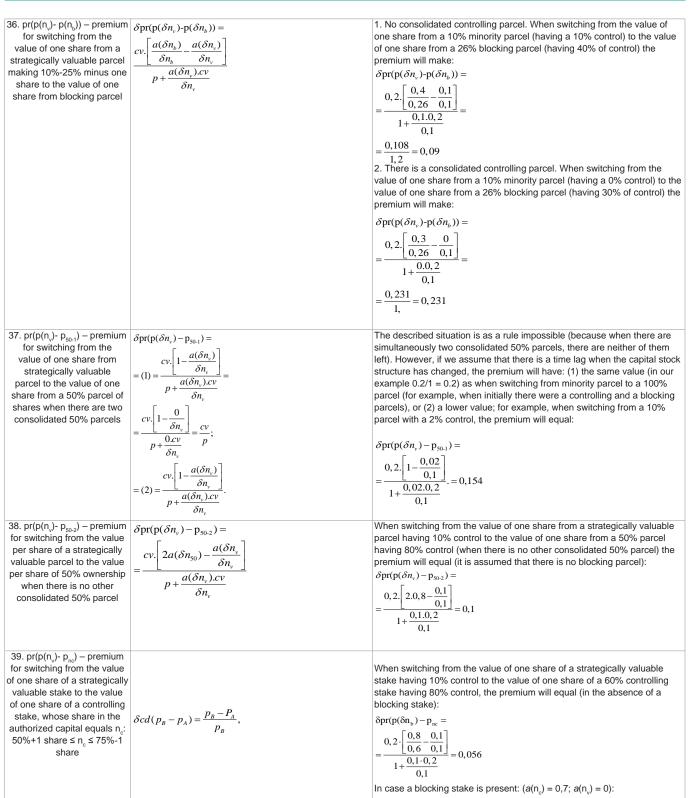p_{100} is calculated value of one share in a 100% parcel of shares;

cv is the value of control calculated per one share expressed in absolute form (money);

Table 2: Equations for calculation of relative premiums for control (calculated per one share) [11].

Statement of the basic and target levels of value [12]	Equations for calculation of relative premiums for control	Sample calculation
29. pr(p-p(n_v)) – premium for switching of the value of one share from minority parcel (making less than 10% of the company capital stock) to the value of one share from strategically valuable parcel making 10% to 25% minus one share.	$$\delta\mathrm{pr(p\text{-}p}(\delta n_v)) = \frac{a(\delta n_v).CV}{n_v.p} =$$ $$= \frac{a(\delta n_v).\delta CV}{n_v} =$$ $$= \frac{a(\delta n_v).cv}{\delta n_v.p} = \frac{a(\delta n_v).\delta cv}{\delta n_v}$$	1.If there are no consolidated controlling and blocking packages, premium for switching from minority parcel to 15% parcel having 25% control part will equal: $$\delta\mathrm{pr(p\text{-}p}(0,15)) =$$ $$= \frac{0,25.0,2}{0,15} = 0,333$$ 2.If there are consolidated controlling and blocking parcels, premium for switching from minority to 15% parcel having 0% control part will equal: $$\delta\mathrm{pr(p\text{-}p}(0,15)) =$$ $$= \frac{0.0,2}{0,15} = 0$$
30. pr(p-p(n_b)) – premium for switching from the value of one share from minority parcel to the value of one share from blocking parcel of shares (from 25% to 50% minus one share)	$$\delta\mathrm{pr(p\text{-}p}(\delta n_b)) = \frac{a(\delta n_b).CV}{n_b.p} =$$ $$= \frac{a(\delta n_b).\delta CV}{n_b} =$$ $$= \frac{a(\delta n_b).cv}{\delta n_b.p} = \frac{a(\delta n_b).\delta cv}{\delta n_b}$$	1. If there is no consolidated controlling package, premium for switching from minority parcel to 26% parcel having 40% control part will equal: $$\delta\mathrm{pr(p\text{-}p}(0,26)) =$$ $$= \frac{0,40.0,2}{0,26} = 0,308$$ 2. If there is consolidated controlling parcel, premium for switching from minority to 26% parcel having 30% control part will equal: $$\delta\mathrm{pr(p\text{-}p}(0,26)) =$$ $$= \frac{0,30.0,2}{0,26} = 0,231$$
31. pr(p-p_{50-1}) – premium for switching from the value of one share from minority parcel to the value of one share from a 50% parcel when there are two consolidated 50% parcels	$$\delta\mathrm{pr(p\text{-}p}_{50\text{-}1}) = \frac{CV}{MC} = cv$$	When switching from the value of one share from minority parcel to the value of one share from a 50% parcel of shares in situation with two 50% parcels premium will equal: $$\delta\mathrm{pr(p\text{-}p}_{50\text{-}1}) = \frac{20}{100} = 0,2$$
32. pr(p-p_{50-2}) – premium for switching of the value of one share from minority parcel to the value of one share from a 50% parcel of shares when there is no other consolidated 50% parcel	$$\delta cd(p_A - p_B) = \frac{\delta cd(p_B - p_A)}{1 - \delta cd(p_B - p_A)},$$	When switching from the value of one share from minority parcel to the value of one share from a 50% parcel of shares having 75% part of control (in situation when there is no other consolidated 50% parcel) the premium will equal: $$\delta\mathrm{pr(p\text{-}p}_{50\text{-}2}) =$$ $$=2.0,75.0,2=0,3$$
33. pr(p-p_{nc}) – premium for switching from the value of one share from minority parcel to the value of one share from controlling parcel (from 50%+1 share to 75% minus one share)	$$\delta\mathrm{pr(p\text{-}p}_{nc}) = \frac{a(\delta n_c).CV}{p.n_c} =$$ $$= \frac{a(\delta n_c).cv}{p.\delta n_c} = \frac{a(\delta n_c).\delta cv}{\delta n_c}$$	When switching from the value of one share from minority parcel to the value of one share from a 60% controlling parcel of shares (having 70% $$\delta\mathrm{pr(p\text{-}p}(0,6)) =$$ $$= \frac{0,7.0,2}{0,6} = 0,233$$
34. pr(p-p_{sc}) – premium for switching from the value of one share from minority parcel to the value of one share from supercontrolling parcel (from 75% to 100% minus one share)	$$\delta\mathrm{pr(p\text{-}p}_{sc}) = \frac{CV}{p.n_{sc}} =$$ $$= \frac{\delta cv}{\delta n_{sc}}$$	When switching from the value of one share from minority parcel to the value of one share from a 75% super controlling parcel of shares the premium will equal: $$\delta\mathrm{pr(p\text{-}p}(0,75)) =$$ $$= \frac{0,2}{0,75} = 0,267$$
35. pr(p-p_{100}) – premium for switching from the value of one share from minority parcel to the value of one share from a 100% parcel	$$\delta\mathrm{pr(p\text{-}p}_{100}) = \frac{CV}{p.N} = \delta cv$$	When switching from the value of one share from minority parcel to the value of one share from a 100% super-controlling parcel of shares the premium will equal: $$\delta\mathrm{pr(p\text{-}p}_{100}) = 0,2$$

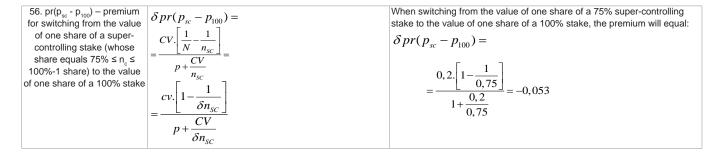|		
36. $pr(p(n_v)-p(n_b))$ – premium for switching from the value of one share from a strategically valuable parcel making 10%-25% minus one share to the value of one share from blocking parcel	$\delta pr(p(\delta n_v)\text{-}p(\delta n_b)) =$ $cv.\dfrac{\left[\dfrac{a(\delta n_b)}{\delta n_b}-\dfrac{a(\delta n_v)}{\delta n_v}\right]}{p+\dfrac{a(\delta n_v).cv}{\delta n_v}}$	1. No consolidated controlling parcel. When switching from the value of one share from a 10% minority parcel (having a 10% control) to the value of one share from a 26% blocking parcel (having 40% of control) the premium will make: $\delta pr(p(\delta n_v)\text{-}p(\delta n_b)) =$ $=\dfrac{0,2.\left[\dfrac{0,4}{0,26}-\dfrac{0,1}{0,1}\right]}{1+\dfrac{0,1.0,2}{0,1}}=$ $=\dfrac{0,108}{1,2}=0,09$ 2. There is a consolidated controlling parcel. When switching from the value of one share from a 10% minority parcel (having a 0% control) to the value of one share from a 26% blocking parcel (having 30% of control) the premium will make: $\delta pr(p(\delta n_v)\text{-}p(\delta n_b)) =$ $=\dfrac{0,2.\left[\dfrac{0,3}{0,26}-\dfrac{0}{0,1}\right]}{1+\dfrac{0.0,2}{0,1}}=$ $=\dfrac{0,231}{1,}=0,231$
37. $pr(p(n_v)-p_{50\text{-}1})$ – premium for switching from the value of one share from strategically valuable parcel to the value of one share from a 50% parcel of shares when there are two consolidated 50% parcels	$\delta pr(p(\delta n_v)-p_{50\text{-}1}) =$ $=(1)=\dfrac{cv.\left[1-\dfrac{a(\delta n_v)}{\delta n_v}\right]}{p+\dfrac{a(\delta n_v).cv}{\delta n_v}}=$ $=\dfrac{cv.\left[1-\dfrac{0}{\delta n_v}\right]}{p+\dfrac{0.cv}{\delta n_v}}=\dfrac{cv}{p};$ $=(2)=\dfrac{cv.\left[1-\dfrac{a(\delta n_v)}{\delta n_v}\right]}{p+\dfrac{a(\delta n_v).cv}{\delta n_v}}.$	The described situation is as a rule impossible (because when there are simultaneously two consolidated 50% parcels, there are neither of them left). However, if we assume that there is a time lag when the capital stock structure has changed, the premium will have: (1) the same value (in our example 0.2/1 = 0.2) as when switching from minority parcel to a 100% parcel (for example, when initially there were a controlling and a blocking parcels), or (2) a lower value; for example, when switching from a 10% parcel with a 2% control, the premium will equal: $\delta pr(p(\delta n_v)-p_{50\text{-}1}) =$ $=\dfrac{0,2.\left[1-\dfrac{0,02}{0,1}\right]}{1+\dfrac{0,02.0,2}{0,1}}.=0,154$
38. $pr(p(n_v)-p_{50\text{-}2})$ – premium for switching from the value per share of a strategically valuable parcel to the value per share of 50% ownership when there is no other consolidated 50% parcel	$\delta pr(p(\delta n_v)-p_{50\text{-}2}) =$ $=\dfrac{cv.\left[2a(\delta n_{50})-\dfrac{a(\delta n_v)}{\delta n_v}\right]}{p+\dfrac{a(\delta n_v).cv}{\delta n_v}}$	When switching from the value of one share from a strategically valuable parcel having 10% control to the value of one share from a 50% parcel having 80% control (when there is no other consolidated 50% parcel) the premium will equal (it is assumed that there is no blocking parcel): $\delta pr(p(\delta n_v)-p_{50\text{-}2}) =$ $=\dfrac{0,2.\left[2.0,8-\dfrac{0,1}{0,1}\right]}{1+\dfrac{0,1.0,2}{0,1}}=0,1$
39. $pr(p(n_v)-p_{nc})$ – premium for switching from the value of one share of a strategically valuable stake to the value of one share of a controlling stake, whose share in the authorized capital equals n_c: 50%+1 share ≤ n_c ≤ 75%-1 share	$\delta cd(p_B-p_A)=\dfrac{p_B-P_A}{p_B},$	When switching from the value of one share of a strategically valuable stake having 10% control to the value of one share of a 60% controlling stake having 80% control, the premium will equal (in the absence of a blocking stake): $\delta pr(p(\delta n_b)-p_{nc} =$ $=\dfrac{0,2\cdot\left[\dfrac{0,8}{0,6}-\dfrac{0,1}{0,1}\right]}{1+\dfrac{0,1\cdot 0,2}{0,1}}=0,056$ In case a blocking stake is present: ($a(n_c)=0,7$; $a(n_v)=0$):

40. pr(p(n_v)- p_SC) – premium for switching from the value of one share of a strategically valuable stake to the value of one share of a super-controlling stake	$\delta pr(p(\delta n_v) - p_{SC}) =$ $\dfrac{cv.\left[\dfrac{1}{\delta n_{SC}} - \dfrac{0}{\delta n_v}\right]}{p + \dfrac{0.cv}{\delta n_v}} = \dfrac{cv}{\delta n_{SC} \cdot p}$	When switching from the value of one share of a strategically valuable stake (which lacks control) to the value of one share of a super-controlling stake (75%), the premium will equal: $\delta pr(p(\delta n_v) - p_{SC} =$ $= \dfrac{0,2}{0,75 \cdot 1} = 0,267$
41. pr(p(n_v)- p_100) – premium for switching from the value of one share of a strategically valuable stake to the value of one share of a 100% stake	$P_c = p_{tc} \cdot N_c,$	When switching from the value of one share of a strategically valuable stake to the value of one share of a 100% stake, the premium will equal (as applied to the cases when a 10% stake has 2% control): $\delta pr(p(\delta n_b) - p_{100} =$ $= \dfrac{0,2 \cdot \left[1 - \dfrac{0,02}{0,1}\right]}{1 + \dfrac{0,02 \cdot 0,2}{0,1}} = 0,154$
42. pr(p(n_b)- p_50-1) – premium for switching from the value of one share of a blocking stake to the value of one share of a 50% stake, in the presence of two 50% consolidated stakes	$\delta pr(p(\delta n_b) - p_{50-1}) =$ $\dfrac{cv.\left[1 - \dfrac{a(\delta n_b)}{\delta n_b}\right]}{p + \dfrac{a(\delta n_b).cv}{\delta n_b}}$	The given situation, as a rule, is impossible (because as soon as there exist two consolidated stakes, no other stake is possible). Nevertheless, if we assume that there is a time lag during which the capital stock structure has changed, when switching from a 26% blocking stake (having 30% control) to the given capital stock structure, the premium will equal: $\delta pr(p(\delta n_b) - p_{50-1} =$ $= \dfrac{0,2 \cdot \left[1 - \dfrac{0,3}{0,26}\right]}{1 + \dfrac{0,3 \cdot 0,2}{0,26}} = -0,025$
43. pr(p(n_b) - p_50-2) – premium for switching from the value of one share of a blocking stake to the value of one share of a 50% stake, in the absence of another 50% consolidated stake	$\delta pr(p(\delta n_b) - p_{50-2}) =$ $\dfrac{cv.\left[2a(\delta n_{50}) - \dfrac{a(\delta n_b)}{\delta n_b}\right]}{p + \dfrac{a(\delta n_b).cv}{\delta n_b}}$	When switching from a 26% stake having 30% control to a 50% stake having 70% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{50-2} =$ $= \dfrac{0,2 \cdot \left[2 \cdot 0,7 - \dfrac{0,3}{0,26}\right]}{1 + \dfrac{0,3 \cdot 0,2}{0,26}} = 0,04$ When switching from a 26% stake having 20% control to a 50% stake having 80% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{50-2} =$ $= \dfrac{0,2 \cdot \left[2 \cdot 0,8 - \dfrac{0,2}{0,26}\right]}{1 + \dfrac{0,2 \cdot 0,2}{0,26}} = 0,144$
44. pr(p(n_b) - p_nc) – premium for switching from the value of one share of a blocking stake to the value of one share of a controlling stake, whose share in the authorized capital equals n_c: 50%+1 share ≤ n_c ≤ 75%-1 share	$\delta pr(p(\delta n_b) - p_{nc}) =$ $\dfrac{cv.\left[\dfrac{a(\delta n_C)}{\delta n_C} - \dfrac{a(\delta n_b)}{\delta n_b}\right]}{p + \dfrac{a(\delta n_b).cv}{\delta n_b}}$	When switching from a 26% stake having 30% control to a 60% stake having 70% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{nc} =$ $= \dfrac{0,2 \cdot \left[\dfrac{0,7}{0,6} - \dfrac{0,3}{0,26}\right]}{1 + \dfrac{0,3 \cdot 0,2}{0,26}} = 0,002$ When switching from a 26% stake having 20% control to a 60% stake having 80% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{nc} =$ $= \dfrac{0,2 \cdot \left[\dfrac{0,8}{0,6} - \dfrac{0,2}{0,26}\right]}{1 + \dfrac{0,2 \cdot 0,2}{0,26}} = 0,098$

45. pr(p(n$_b$)- p$_{sc}$) – premium for switching from the value of one share of a blocking stake to the value of one share of a super-controlling stake	$\delta\mathrm{pr}(p(\delta n_b) - p_{SC}) =$ $cv.\dfrac{\left[\dfrac{1}{\delta n_{SC}} - \dfrac{a(\delta n_b)}{\delta n_b}\right]}{p + \dfrac{a(\delta n_b).cv}{\delta n_b}}$	When switching from a 26% having 30% control to a 75% stake having 100% control, the premium will equal: $\delta\mathrm{pr}(p(\delta n_b) - p_{SC}) =$ $= \dfrac{0,2 \cdot \left[\dfrac{1}{0,75} - \dfrac{0,3}{0,26}\right]}{1 + \dfrac{0,3 \cdot 0,2}{0,26}} = 0,029$ When switching from a 26% stake having 20% control to a 75% stake having 100% control, the premium will equal: $\delta\mathrm{pr}(p(\delta n_b) - p_{SC}) =$ $= \dfrac{0,2 \cdot \left[\dfrac{1}{0,75} - \dfrac{0,2}{0,26}\right]}{1 + \dfrac{0,2 \cdot 0,2}{0,26}} = 0,098$
46. pr(p(n$_b$)- p$_{100}$) – premium for switching from the value of one share of a blocking stake to the value of one share of a 100% stake	$\delta\mathrm{pr}(p(\delta n_b) - p_{100}) =$ $= \dfrac{cv.\left[1 - \dfrac{a(\delta n_b)}{\delta n_b}\right]}{p + \dfrac{a(\delta n_b).cv}{\delta n_b}}$	When switching from a 26% stake having 30% control to a 100% stake having 100% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{100}) =$ $= \dfrac{0,2.\left[1 - \dfrac{0,3}{0,26}\right]}{1 + \dfrac{0,3.0,2}{0,26}} = -0,025$ When switching from a 26% stake having 20% control to a 100% stake having 100% control, the premium will equal: $\delta pr(p(\delta n_b) - p_{100}) =$ $\dfrac{0,2.\left[1 - \dfrac{0,2}{0,26}\right]}{1 + \dfrac{0,2.0,2}{0,26}} = 0,04$
47. pr(p$_{50-1}$ - p$_{50-2}$) – premium for switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 50% stake, in the absence of another 50% consolidated stake	$\delta pr(p_{50-1} - p_{50-2}) =$ $= \dfrac{CV.[2a(\delta n_{50-2}) - 1]}{MC_C} =$ $= \dfrac{CV.[2a(\delta n_{50-2}) - 1]}{p_{100}}$	When switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 50% stake, in the absence of another 50% consolidated stake, and when the 50% stake has 75% control, the premium will equal: $\delta pr(p_{50-1} - p_{50-2})$ $= \dfrac{0,2.[2.0,75 - 1]}{1,2} = 0,083$
48. pr(p$_{50-1}$ - p$_{nc}$) – premium for switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a controlling stake, whose share in the authorized capital equals n$_c$: 50%+1 share ≤ n$_c$ ≤ 75%-1 share	$\delta pr(p_{50-1} - p_{nc})$ $= \dfrac{CV.\left[\dfrac{a(\delta n_c)}{n_c} - \dfrac{1}{N}\right]}{p_{100}}$ $= \dfrac{cv.\left[\dfrac{a(\delta n_c)}{\delta n_c} - 1\right]}{p_{100}}$	When switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 60% stake having 80% control, the premium will equal: $\delta pr(p_{50-1} - p_{nc}) =$ $= \dfrac{0,2.\left[\dfrac{0,8}{0,6} - 1\right]}{1,2} = 0,056$
49. pr(p$_{50-1}$ - p$_{sc}$) – premium for switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a super-controlling stake	$\delta pr(p_{50-1} - p_{sc})$ $= \dfrac{CV.\left[\dfrac{1}{n_{sc}} - \dfrac{1}{N}\right]}{p_{100}}$ $= \dfrac{cv.\left[\dfrac{1}{\delta n_{SC}} - 1\right]}{p_{100}}$	When switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 95% stake, the premium will equal: $\delta pr(p_{50-1} - p_{sc}) =$ $= \dfrac{0,2.\left[\dfrac{1}{0,95} - 1\right]}{1,2} = 0,009$
50. pr(p$_{50-1}$ - p$_{100}$) – premium for switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 100% stake	$\delta pr(p_{50-1} - p_{100}) = 0$	When switching from the value of one share of a 50% stake, in the presence of two 50% consolidated stakes, to the value of one share of a 100% stake, the premium will equal: $\delta pr(p_{50-1} - p_{100}) = 0$

51. pr(p_{50-2} - p_{nc}) – premium for switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake, to the value of one share of a controlling stake, whose share in the authorized capital equals: $_{nc}$: 50%+1 share ≤ n_c ≤ 75%-1 share	$\delta pr(p_{50-2} - p_{nc}) =$ $= \dfrac{cv.\left[\dfrac{a(\delta n_c)}{\delta n_c} - 2a(\delta n_{50-2})\right]}{p + 2a(\delta n_{50-2}).cv}$	When switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake (and its owner holds a 70% control ownership interest), to the value of one share of a 60% controlling stake having 85% control, the premium will equal: $\delta pr(p_{50-2} - p_{nc}) =$ $= \dfrac{0,2.\left[\dfrac{0,85}{0,6} - 2.0,75\right]}{1 + 2.0,75.0,2} = -0,013$
52. pr(p_{50-2} - p_{sc}) – premium for switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake, to the value of one share of a super-controlling stake	$\delta pr(p_{50-2} - p_{sc}) =$ $= \dfrac{cv.\left[\dfrac{1}{\delta n_{sc}} - 2a(\delta n_{50-2})\right]}{p + 2a(\delta n_{50-2}).cv}$	When switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake (and its owner holds a 75% control ownership interest), to the value of one share of a 75% super-controlling stake, the premium will equal: $\delta pr(p_{50-2} - p_{sc}) =$ $= \dfrac{0,2.\left[\dfrac{1}{0,75} - 2.0,75\right]}{1 + 2.0,75.0,2} = -0,025$
53. pr(p_{50-2} - p_{100}) – premium for switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake, to the value of one share of a 100% stake	$\delta pr(p_{50-2} - p_{100}) =$ $= \dfrac{cv.[1 - 2a(\delta n_{50-2})]}{p + 2a(\delta n_{50-2}).cv}$	When switching from the value of one share of a 50% stake, in the absence of another 50% consolidated stake (and its owner holds a 75% control ownership interest), to the value of one share of a 100% super-controlling stake, the premium will equal: $\delta pr(p_{50-2} - p_{100}) =$ $= \dfrac{0,2.[1 - 2.0,75]}{1 + 2.0,75.0,2} = -0,077$
54. pr(p_{nc} - p_{sc}) – premium for switching from the value of one share of a controlling stake (whose share equals 50%+1 share ≤ n_c ≤ 75%-1 share) to the value of one share of a super-controlling stake	$\delta pr(p_{nc} - p_{sc}) =$ $= \dfrac{cv.\left[\dfrac{1}{\delta n_{sc}} - \dfrac{a(\delta n_c)}{\delta n_c}\right]}{p + \dfrac{a(\delta n_c).cv}{\delta n_c}}$	When switching from the value of one share of a 60% controlling stake (having 75% control), to the value of one share of a 75% super-controlling stake, the premium will equal: $\delta pr(p_{nc} - p_{sc}) =$ $= \dfrac{0,2.[\dfrac{1}{0,75} - \dfrac{0,75}{0,6}]}{1 + \dfrac{0,75.0,2}{0,6}} = 0,013$
55. pr(p_{nc} - p_{100}) – premium for switching from the value of one share of a controlling stake (whose share equals 50%+1 share ≤ n_c ≤ 75%-1 share) to the value of one share of a 100% stake	$\delta pr(p_{nc} - p_{100}) =$ $= \dfrac{cv.\left[1 - \dfrac{a(\delta n_c)}{\delta n_c}\right]}{p + \dfrac{a(\delta n_c).cv}{\delta n_c}}$	When switching from the value of one share of a 60% controlling stake (having 70% control), to the value of one share of a 100% super-controlling stake, the premium will equal: $\delta pr(p_{nc} - p_{100}) =$ $= \dfrac{0,2.\left[1 - \dfrac{0,7}{0,6}\right]}{1 + \dfrac{0,7.0,2}{0,6}} = -0,027$ *E.g.,* Under the aforementioned values of all the other parameters, the value of one share of a 60% stake having 70% control will equal 1+ 0.7×0.2/0.6 = 1.233. The value of one share of a 100% stake under the aforementioned parameters will equal 120/100 = 1.2. Thus, the premium for switching from the value of one share of a 60% stake to the value of one share of a 100% stake will equal (1.2 – 1.233)/1.233 = -0.027, which is the same as for the calculated value given hereinbefore. *Note* 1. When returning from the value of one share of a 100% super-controlling stake to the value of one share of a 60% stake (having 70% control), the premium will equal: $\delta pr(p_{100} - p_{nc}) =$ $= \dfrac{20.(0,7 - 0,6)}{0,6.120} = 0,028$ $\dfrac{CV.\left(a(\delta n_c) - \delta(n_c)\right)}{\delta(n_c).MC_C}$ *Note* 2. If we compare the determined values, the premium to be applied will equal 2.8% when switching from 1.2 to 1.233; but in case of return, a 2.7% discount shall be utilized. It should be underlined that if we know the greater of two values (i.e., the premium for control = 2.8%), we can define the lower value (i.e., the discount for lack/decrease of control = 2.7%) using aforementioned formula (13).

56. pr(p_{sc} - p_{100}) – premium for switching from the value of one share of a super-controlling stake (whose share equals 75% ≤ n_c ≤ 100%-1 share) to the value of one share of a 100% stake	$$\delta pr(p_{sc} - p_{100}) = \\ = \frac{CV.\left[\frac{1}{N} - \frac{1}{n_{sc}}\right]}{p + \frac{CV}{n_{sc}}} = \\ = \frac{cv.\left[1 - \frac{1}{\delta n_{sc}}\right]}{p + \frac{CV}{\delta n_{sc}}}$$	When switching from the value of one share of a 75% super-controlling stake to the value of one share of a 100% stake, the premium will equal: $$\delta pr(p_{sc} - p_{100}) = \\ = \frac{0,2.\left[1 - \frac{1}{0,75}\right]}{1 + \frac{0,2}{0,75}} = -0,053$$

N is total quantity of the company shares (in the example $N = 100$),

cv is the value of control expressed in relative form [6].

Between MC_c and MC there are obviously following dependences:

$$MC_c = MC \times (1 + pr_{p-p100}), \quad (4)$$

$$MC = MC_c \times (1 - cdp_{100-p}), \quad (5)$$

$$P_{100} = p \times (1 + \delta pr_{p-p100}), \quad (6)$$

$$P = p_{100} \times (1 + \delta cd_{p100-p}) \quad (7)$$

where pr_{p-p100} is relative value of the premium for the control over 100% of the parcel of shares (or 100% of interest in the capital stock) when switching from the value of one share from the parcel having no powers of control to the value of one share from a 100% parcel,

cd_{p100-p} is relative discount for the lack of control when switching from the value of one share as part of a 100% parcel to the value of one share from the parcel having no powers of control.

Apparent equations result from (4) ÷ (7):

$$\delta pr_{p-p100} = \frac{MC_c}{MC} - 1 = \frac{P_{100}}{p} - 1 = \frac{120}{100} - 1 = \frac{1,2}{1} - 1 = 0,2 \quad (8)$$

$$\delta cd_{p100-p} = 1 - \frac{MC}{MC_c} = 1 - \frac{p}{p_{100}} = 1 - \frac{100}{120} = 1 = \frac{1}{1,2} = 0,167, \quad (9)$$

$$\delta cd_{p-p100} = \frac{\delta cd_{p100-p}^c}{1 - \delta cd_{p100-p}} = \frac{0,167}{1 - 0,167} = 0,2, \quad (10)$$

$$\delta cd_{p100-p} = \frac{\delta pr_{p-p100}}{1 + \delta pr_{p-p100}} = \frac{0,2}{1 + 0,2} = 0,167. \quad (11)$$

Generally speaking, control premium or discount can be provided in absolute and relative form (calculated per the value of one share):

control premium/discount for the lack of control in absolute form calculated per one share (cv) is an absolute difference expressed in money equivalent between the value of one share in two parcels having different controlling powers (at that the absolute values of premiums and discounts are equal in magnitude, but opposite in sign); in relative form control premium/discount for the lack of control is a relative difference between the value of one share in two parcels having different controlling powers.

Laws on joint-stock companies provide different powers for the owners of different parcels of shares. The value of such parcels most of all differs in the value calculated on direct relation (as an interest in the company capital stock). For further consideration and analyses we shall identify the corresponding levels of the value of one share

depending on its relation to one of the following parcels (Table 1).

Table 1 presents eight levels of value for which there are 28 possible switches (premiums) from a lower level to another higher level of value, and there is the same quantity (28) switches (discounts) from a higher level to a lower value level. Taking into account the notations given hereinabove further on us will use the following notations system for consideration of premiums and discounts:

1. pr(p_A-p_B) – premium for switching from the value of one share from parcel A to the value of one share from parcel B (for example, $pr(p(nb)-pnc)$ is a premium for switching from the value of one share from a blocking parcel (from 25%+1 share) to the value of one share from a controlling (from 50%+1 share to 75% minus 1 share) parcel);

2. cd(p_B-p_A) – premium for reduction in control when switching from the value of one share from parcel B to the value of one share from parcel A (for example, $cd(p_{sc}-p_{nc})$ is a discount for switching from the value of one share from a super controlling parcel (from 75% to 100%) to the value of one share from a controlling parcel having 50%+1 < n_c ≤ 75%).

It is noteworthy that generally speaking if we know the value of one share from a certain parcel of shares (pA) and we need to determine the value of one share from another parcel of shares (pB) having more powers of control, the calculation can be done as follows:

$$p_B = p_A . (1 + \delta pr(p_A - p_B)) \quad (12)$$

Where pr(p_A-p_B) is relative premium for the larger level of control of the shareholders owning parcel B as compared to the level of control of the shareholders owing parcel B.

Similarly, if we know the value of one share from a certain parcel of shares (pB) and it is necessary to estimate the value of one share from another parcel of shares (pA) having less power of control, the calculation can be done as follows:

$$p_A = p_B . (1 + \delta cd(p_B - p_A)) \quad (13)$$

where cd(p_B-p_A) is relative discount for the lower power of control of the shareholders owing parcel A as compared to the power of control of the shareholders owing parcel B.

It is noteworthy that mathematics of premiums and discounts like operations with indices should have a chain character: premium for switching from level A (value of a share from parcel A) to level C (value of a share from parcel C) consists of two premiums — a premium for switching from level A to level B (value of a share from parcel B, levels: A<B<C) and a premium for switching from level B

Table 3: Results of Control Premium Study published by the Factset Mergerstat.

	Including Negative Premiums		Excluding Negative Premiums	
	# of Deals	Premium	# of Deals	Premium
Domestic Average	49	34.4%	46	37.8%
Domestic Median	49	30.4%	46	32.8%
International Average	85	15.9%	60	36.0%
International Median	85	18.4%	60	27.0%
Overall Average	134	22.6%	106	36.7%
Overall Median	134	21.8%	106	28.6%
p_{100}	- value of one share from a 100% parcel of shares			

to level C:

$$\delta pr(p_A - p_C) = [(1 + \delta pr(p_A - p_B)).(1 + \delta pr(p_B - p_C))] - 1$$

(14)

For example, a premium for switching from value of share from a minority parcel (making less than 10% of the capital stock of the company) to the value of one share from a controlling parcel (from 50%+1 share to 75% minus one share) when we know the premiums pr(p-p(nb)) and pr(p(nb)-pnc) can be calculated as follows (Table 1):

$$\delta pr(p - p_{nc}) = [(1 + \delta pr(p - p(\delta n_b))).(1 + \delta pr(p(\delta n_b - p_{nc})))] - 1$$

In elaboration of (10)-(11) the relations between relative values of premiums and discounts for the same level of switches can generally be given as follows:

$$\delta cd(p_B - p_A) = \frac{\delta pr(p_A - p_B)}{1 + \delta pr(p_A - p_B)},$$
(15)

$$\delta cd(p_A - p_B) = \frac{\delta cd(p_B - p_A)}{1 - \delta cd(p_B - p_A)},$$
(16)

As for the relation between absolute values of premiums and bonuses (for the same level of switches), it can be given as follows:

$$cd(p_B - p_A) = - pr(p_A - p_B)$$
(17)

where $cd(p_B - p_A)$ is absolute value (expressed in money) of the discount for the reduction in control when switching from the value of one share from parcel B to the value of one share from parcel A;

$pr(p_A - p_B)$ is absolute value (expressed in money) of the discount for the increase in control when switching from the value of one share from parcel A to the value of one share from parcel B.

Sign «-» (minus) in (17) denotes the opposite direction of impact on the value of the discount for the lack of control if compared to the premium for the presence of control. In equations (4) and (5) the signs already account for this aspect (difference in the direction of impact on the value of the share), so when using this calculation of the discount for the lack of control (according to equation (5)) in practice we should use the modulus of its value.

When estimating the capital stock one should take into account the cost of the stock option plans (if the company has the practice of manager's participation in profits that is fixed in the contracts concluded between the company and the top managers). We will further on bear in mind that values MC and MC_c already account for this factor.

There is another indirect method to determine the value of control that is based on the use of information on the costs of the bids for the shares from controlling or blocking parcels of shares. This method will be considered in the sixth part of this article.

For further analysis we will proceed from the premises described in [1], namely:

1.   Simple increase of the quantity of shares in the parcel gives an advantage (quantity factor).

2.   Increase of minority parcel of shares decreases the advantages of other similar packages (redistribution factor).

3.   Additional advantages of the controlling parcel of shares (> 50%) area obtained at the expense of reduction of advantages of minority parcels (balance factor).

4.   A group of combined parcels of shares has advantages [7] that at least equal the sum of the advantages of each of the parcels (combination factor).

5.   Increase in the quantity of small parcels of shares that are similar to each other in their interest when there is no controlling parcel of shares decreases the total premium value (dispersal factor).

6.   Approaching of two or more parcels of shares to the controlling parcel escalates the competition and increases the advantages of small minority parcels (competition factor).

Taking into account these introductory ideas and agreed notations we can proceed to the description of the methods for estimation of the values of shares from different parcels and to the premiums and discounts related to such estimations.

Estimation of the value of one share from different parcels

On the basis of the premises and numbers above it can be concluded that the value of one share from minority parcel (making less than 10% of the company capital share) will make [8]:

$$p = \frac{MC}{N} = \frac{MC_c}{N}.(1 - \delta cd(p_{100} - p)) = \frac{100}{100} = \frac{120}{100}.(1 - 0,167) = 1, \ (18)$$

Where all notations correspond to those accepted above.

One share from strategically valuable parcel (making 10% to 25% minus one share) will have the same value if there is one consolidated super-controlling parcel.

If there is no one consolidated super-controlling parcel, the value of one share from strategically significant parcel (making 10% to 25%

Table 4: Analysis of shareholders' rights in accordance with the size of a stake (due to the current edition of Order 208-FZ "On Joint-Stock Companies") [16].

#	Description of rights affecting the cost	Minimum size of the stake ensuring an appropriate right (% of voting shares)
1	The right to demand an extraordinary general meeting of shareholders	10
2	The right to demand an audit check of the financial and economic activities of the company	10
3	Right of access to documents [accounting and reports of the meetings of the KIO]	25
4	The right to block approval of issues requiring 75% of the vote	от 25%+1
5	Provision of a quorum at a re-meeting of shareholders	30
6	Determination of the number of members of the board of directors (supervisory board of the company), election of its members and early termination of their powers	50%+1
7	Increase of the authorized capital of a company by increasing the nominal value of shares or by placing additional shares	50%+1
8	Decrease in the authorized capital by acquisition of a part of shares by the company in order to reduce their total number, as well as by paying off shares acquired or bought back by the company	50%+1
9	Establishment of the executive body of the company, early termination of its authority	50%+1
10	Election of members of the Company's Audit Commission and early termination of their powers	50%+1
11	Approval of the company's auditor	50%+1
12	Approval of annual reports, annual financial statements, including profit and loss statements of the company, as well as distribution of profits, including payment (declaration) of dividends	50%+1
13	Determination of the procedure for holding a general meeting of shareholders	50%+1
14	Election of members of the counting commission of the company and early termination of their powers	50%+1
15	Splitting and consolidation of shares	50%+1
16	Making decisions on the approval of major transactions, the subject of which is property with a value accounting for 25 to 50% of the book value, in case of unavailability of the Board of Directors of unanimity and their decision on discussing the issue at general meeting (p. 2, cl. 2, cl. 79)	50%+1
17	Making a decision on participation in holding companies, financial and industrial groups, associations and other associations of commercial organizations	50%+1
18	Approval of internal documents regulating the activities of the company's bodies	50%+1
19	Amendments and additions to the company's charter or approval of the company's charter in a new edition	75
20	Reduction of the authorized capital of the company by reducing the nominal value of shares	75% (p. 3, cl. 29)
21	Reorganization of the company	75
22	Liquidation of the company, appointment of the liquidation commission and approval of the interim and final liquidation balance sheets	75
23	Determination of the number, nominal value, category (type) of authorized shares and the rights granted by these shares	75
24	Acquisition of outstanding shares by the company	75
25	Making a decision on the approval of a major transaction, the subject of which is property, the value of which is more than 50% of the book value of the assets	75
26	Placement of shares (equity securities of the company convertible into shares) by private subscription	75
27	Placement of shares (equity securities of the company convertible into shares) that constitute more than 25% of previously placed ordinary shares through open subscription	75

minus one share) will increase [9] and will equal:

$$p(\delta n_v) = \frac{MC}{N} + \frac{\left(1 - \sum_{i-b}^{e} a(i)\right).CV}{n_v} = p + \frac{a(\delta n_v).CV}{n_v} = p + \frac{a(\delta n_v).cv}{\delta n_v}, \quad (19)$$

where a(i) is control interest accrueing to the owner of the i parcel of shares (0 ≤ a(i) ≤1);

lower index «b» denotes attribution to the blocking parcel of shares;

upper index «c» denotes attribution to the controlling parcel of shares;

a(nv) is the control of the owner of the evaluated parcel of shares making n_v in the issuing company capital stock;

n_v is the quantity of shares in the evaluated stock (pieces/items);

n_v is the company capital stock part belonging to the owner of the evaluated parcel of shares ($\delta n_v = n_v /N$);

the other notations correspond to those accepted above.

Approximate estimation of the a(n_v) control falling to the owner of the evaluated parcel of shares can be performed as follows:

$$a(\delta n_v) \approx \frac{\delta n_v}{\delta n_c + \sum \delta n_{v \, \text{Rest}}}, \quad (20)$$

or (if there is no consolidated controlling parcel):

$$a(\delta n_v) \approx \frac{\delta n_v}{\delta n_b + \sum \delta n_{v \, \text{Rest}}}, \quad (21)$$

where nc is the part of the capital stock of the company belonging to the owner of the controlling parcel (if there is any) (nc = nc /N), nb is the part of the capital stock of the company belonging to the owner of the blocking parcel (nb = nb /N), $\sum \delta n_v$ Rest are aggregated parts of the other (apart from the controlling and blocking parcels) valuable consolidated parcels of shares, the other notations correspond to those accepted above.

Table 5: Points assigned to powers.

#	Description of rights affecting the cost	Minimum size of the stake ensuring an appropriate right (% of voting shares)	The importance (value) of each right (points) (subjective assessment)
1	The right to demand an extraordinary general meeting of shareholders	10	1
2	The right to demand an audit check of the financial and economic activities of the company	10	1 (The sum of points for the holder of a consolidated stake of 10% to 25% -1 = 2)
3	Right of access to documents [accounting and reports of the meetings of the KIO]	25	2 (The sum of points for the holder of a consolidated stake of 25% = 4)
4	The right to block approval of issues requiring 75% of the vote	от 25%+1	3 (The sum of points for the holder of a consolidated stake of 25% + 1 to 30% -1 = 7)
5	Provision of a quorum at a re-meeting of shareholders	30	2 (The sum of points for the owner of a consolidated stake of 30% to 50% = 9)
6	Determination of the number of members of the board of directors (supervisory board of the company), election of its members and early termination of their powers	50% + 1	3
7	Increase of the authorized capital of a company by increasing the nominal value of shares or by placing additional shares	50% + 1	1
8	Decrease in the authorized capital by acquisition of a part of shares by the company in order to reduce their total number, as well as by paying off shares acquired or bought back by the company	50% + 1	1
9	Establishment of the executive body of the company, early termination of its authority	50% + 1	3
10	Election of members of the Company's Audit Commission and early termination of their powers	50% + 1	3
11	Approval of the company's auditor		2
12	Approval of annual reports, annual financial statements, including profit and loss statements of the company, as well as distribution of profits, including payment (declaration) of dividends	50% + 1	3
13	Determination of the procedure for holding a general meeting of shareholders	50% + 1	3
14	Election of members of the counting commission of the company and early termination of their powers	50% + 1	4
15	Splitting and consolidation of shares	50% + 1	1
16	Making decisions on the approval of major transactions, the subject of which is property with a value accounting for 25 to 50% of the book value, in case of unavailability of the Board of Directors of unanimity and their decision on discussing the issue at general meeting (p. 2, cl. 2, cl. 79)	50% + 1	4
17	Making a decision on participation in holding companies, financial and industrial groups, associations and other associations of commercial organizations	50% + 1	4
18	Approval of internal documents regulating the activities of the company's bodies	50% + 1	3 (The sum of points for the holder of a consolidated package of 50% + 1 to 75% -1 = 44)
19	Amendments and additions to the company's charter or approval of the company's charter in a new edition	75	4
20	Reduction of the authorized capital of the company by reducing the nominal value of shares	75% (п. 3 ст. 29)	1
21	Reorganization of the company	75	5
22	Liquidation of the company, appointment of the liquidation commission and approval of the interim and final liquidation balance sheets	75	5
23	Determination of the number, nominal value, category (type) of authorized shares and the rights granted by these shares	75	1
24	Acquisition of outstanding shares by the company	75	1
25	Making a decision on the approval of a major transaction, the subject of which is property, the value of which is more than 50% of the book value of the assets	75	3
26	Placement of shares (equity securities of the company convertible into shares) by private subscription	75	4
27	Placement of shares (equity securities of the company convertible into shares) that constitute more than 25% of previously placed ordinary shares through open subscription	75	4 (The sum of points for the holder of a consolidated stake of 75% = 72)

It is noteworthy that if these are consolidated controlling and blocking parcels or one super-controlling (absolutely controlling) parcel (≥75% of the capital stock), strategic parcels actually seize being strategic (because in such cases their owners actually have no control ($a(n_v) = 0$)), and consequently, the value of one share in such parcels approaches the value of one share from minority parcels of shares.

A more accurate estimation of the control (a) of the owner of the evaluated parcel of shares can be performed by means of a detailed analysis of the capital stock structure taking into account the distribution of all consolidated and legally significant parcels (with account of the powers provided to their owners by current laws), existing alliances and distribution of the places in the Board of Directors of the issuing company. The example of such analysis is given in the fourth part of the article.

Similarly, the value of one share from the blocking parcel of shares (from 25% to 50%) will make:

$$p(\delta n_b) = \frac{MC}{N} + \frac{(1 - a(\delta n_c)).CV}{n_b} = p + \frac{a(\delta n_b).CV}{n_b} = p + \frac{a(\delta n_b).cv}{\delta n_b},$$

$$(22)$$

where a(nc) is the control of the owner of the controlling parcel of shares (nc ≤ a(nc) ≤1; ncmin = 0,51);

a(nb) is the control of the owner of the evaluated blocking parcel of shares (nb) making δnb part in the capital stock of the issuing company;

nb is the quantity of shares in the evaluated blocking parcel (pieces/items);

nb is the part of the company capital stock belonging to the owner of the evaluated blocking parcel (nb = nb /N).

Approximate estimation of control a(nb) of the owner of the evaluated blocking parcel of shares can be performed as follows:

$$a(\delta n_b) \approx \frac{\delta n_b}{\delta n_c + \delta n_b},$$

$$(23)$$

or (if there is no consolidated controlling parcel)

$$a(\delta n_b) \approx \frac{\delta n_b}{\delta n_b + \sum \delta n_{v\,Re\,st}},$$

$$(24)$$

where n_b is the part of the company capital stock belonging to the owner of the blocking parcel (nb = nb /N),

n_c is the part of the company capital stock belonging to the owner of the controlling parcel (nc = nc /N),

$\sum n_v$ Rest are aggregated parts of the other (apart from blocking parcels) valuable consolidated parcels of shares, the other notations correspond to those accepted above.

The value of one share from a 50% parcel when there are two consolidated 50% parcels will equal the value of one share from another consolidated 50% parcel, so its value shall equal:

$$p_{50-1} = \frac{MC_c}{N} = p_{100},$$

$$(25)$$

where p100 is the value of one share from a 100% parcel,

the other notations correspond to those accepted above.

The value of one share from a 50% parcel when there is no other (apart from the evaluated one) consolidated 50% parcel will equal:

$$p_{50-2} = \frac{MC}{N} + \frac{a.CV}{n_{50}} = p + \frac{2a.CV}{N} = p + 2a.cv,$$

$$(26)$$

where a is control part of the evaluated 50% parcel when there is no other (similar) consolidated parcel, n50 is the quantity of shares in the evaluated 50% parcel (equaling half of all shares), the other notations correspond to those accepted above.

The value of one share from the controlling parcel (from 50%+1 share to 75% minus one share) will equal:

$$p_{nc} = \frac{MC}{N} + \frac{a(\delta n_c).CV}{n_c} = p + \frac{a(\delta n_c).cv}{\delta n_c},$$

$$(27)$$

where a(nc) is control part of the owner of the controlling parcel of shares (nc ≤ a(nc) ≤1; nc min = 0.51), making nc part in the issuing company capital stock; nc is the quantity of shares in the evaluated controlling parcel (pieces/items); nc is part in the company capital stock belonging to the owner of the evaluated controlling parcel (nc = nc /N).

Approximate estimation of the control part a(nc) of the owner of the evaluated controlling parcel of shares can be performed as follows:

$$a(\delta n_c) \approx \frac{\delta n_c}{\delta n_c + \delta n_b},$$

$$(28)$$

or (if there is no consolidated blocking parcel)

$$a(\delta n_c) \approx \frac{\delta n_c}{\delta n_c + \sum \delta n_{v\,Re\,st}},$$

$$(29)$$

where n_c is the part of the company capital stock belonging to the owner of the controlling parcel (n_c = n_c /N);

$\sum \delta n_v$ Rest are aggregated parts of the other (apart from the controlling parcel) valuable parcels of shares; the other notations correspond to those accepted above.

The value of one share from the supercontrolling parcel (from 75% to 100% minus one share) equals:

$$p_{sc} = \frac{MC}{N} + \frac{CV}{n_{sc}} = p + \frac{cv}{\delta n_{sc}},$$

$$(30)$$

where nsc is the quantity of shares in the supercontrolling parcel (pieces/items); nsc is the part of the company capital stock belonging to the owner of the supercontrolling parcel (nsc = nsc /N); the other notations correspond to those accepted above.

It may be shown that equation (27) can be added and reduced as follows:

$$p_{sc} = \frac{p_{100}.N - p.(N - n_{sc})}{n_{sc}} = \frac{N.(p_{100} - p) + p.n_{sc}}{n_{sc}} = p + \frac{cv}{\delta n_{sc}}.$$

$$(31)$$

The first equation is derived from the following condition:

$$MC_c = p_{sc}.n_{sc} + p.(N - n_{sc}) = p_{100}.N$$

$$(32)$$

(i. e. the value of shares from a 100% parcel on the one hand equals the sum of the values of shares from the super controlling and minority parcels, from the other hand – to product of the proportionate value of one share from a 100% parcel by the total quantity of all shares).

It is noteworthy that equations (21), (25) and (27) show that

the value of one share from a blocking, controlling and/or super controlling parcels reaches its maximum with minimum quantity of shares in the parcel of that class (i. e. with minimum possible blocking, minimum possible controlling and/or super controlling parcels [10].

The value of one share in a 100% parcel equals:

$$p_{100} = \frac{MC_c}{N} = \frac{MC}{N} + \frac{CV}{N} = p + Cv, \qquad (33)$$

Where all notations correspond to those accepted above.

Calculation of premiums and discounts for the presence and lack of control

Now we can consider the methods of calculation of premiums for increase of control and discount for reduction of control or for its lack.

Absolute value of control as calculated per one share is calculated as a difference between the value of one share as part of a parcel with an increase level of control (pB) and the value of a share as part of a parcel with a reduced level of control or its lack (pA):

$$\mathrm{pr}(p_A\text{-}p_B) = p_B \text{ - } p_A, \qquad (34)$$

where $\mathrm{pr}(p_A\text{-}p_B)$ is an absolute value of the premium for the control when switching from the level of value of share p_A to the level of value of share p_B.

In order to receive the absolute value of the premium for increased control it is necessary to make calculations by formula (31), inserting the corresponding values of p_A and p_B, the equations for which we have obtained in the previous part of the article. According to (Table 1) we are considering a total of eight levels of value. So, there are seven values of premiums corresponding to switches between the neighboring levels of value (from the lower one to the higher one) and 21 values of premiums corresponding to not neighboring switches, i. e. there are 28 absolute values of premiums for increased control for all eight levels of value, and the same quantity of absolute values of discounts for reduced control.

Relative value of control is calculated as a difference between the value of one share from the parcel with an increased level of control (pB) and the value of the share from the parcel with a reduced level of control or its lack (pA) divided by the value of one share from the parcel with a reduced level of control (or its lack):

$$\delta pr(p_A - p_B) = \frac{p_B - p_A}{p_A}, \qquad (35)$$

where $\mathrm{pr}(p_A\text{-}p_B)$ is relative value of the premium for control when switching from level of value of share p_A to the level of value of share p_B.

In order to receive relative value of the premium for increased control it is necessary to make calculations by formula (32) inserting in it the corresponding values of p_A and p_B the equations for which have been received in the previous part of the article. Similarly, following (Table 1) we are considering eight levels of value. Thus, there are seen values of relative premiums corresponding to the switches between neighbouring levels of value (from the lower one to the higher one) and also 21 values of premiums corresponding to not neighbouring switches, i. e. for eight levels there are 28 relative values of the premium for reduced control. Equations (15) – (16) provide the dependence between relative values of the premiums

and discounts for control, at that, relative values of discounts for reduction in control can be generally calculated as follows:

$$\delta cd(p_B - p_A) = \frac{p_B - P_A}{p_B}, \qquad (36)$$

where all notations correspond to those accepted above.

The table below (having its own numbering of the formulae) provide the resulting equations for calculation of relative values of the premiums for control between 8 levels of value after their insertion into formula (32) and required developments. All notations in the table correspond to those accepted above.

It should be noted that depending on the real allocation of the controlling interest share, the value per share of a controlling stake can be either higher or lower than the value per share of a blocking stake. This depends on which of the two ratios of the exact proportion of the "control due" is deviated.

For example, if the number of shares of the controlling stake contains 51 units and the blocking stake contains 26 units (the total number of shares outstanding is 100 units), the exact controlling interest share of the controlling stake, when both stakes have equal values of each share, will be defined as follows:

$$a_{np} = \frac{N_c}{N_c + N_b} = \frac{51}{51 + 26} = 0,662. \qquad (37)$$

If the controlling interest share of a controlling stake is greater than this value, the value of one share of the stake will be greater than the value of one share of a blocking stake, and vise versa. If one holds a controlling stake "50% + 1 share", its further build-up will make sense only for achieving the level of minimum amount of shares of a super-controlling stake accounting for but not exceeding 75% of the authorized capital [13].

Taking into account these circumstances, a behavior of large players in the stock market becomes clear -the drive to hold a minimum significant (controlling/super-controlling) stake. For example, when acquiring a 51% stake, its further build-up to the next strategic level of 75% will lead to a lower "specific value of the parcel" (i.e., lower value of one share of the stake). When switching from "75% -1 share" to 75%, the value of each share of the stake tends to increase. This is one of the types of business run by investment companies, including buybacks of outstanding shares at minority prices, their merging into strategic stakes, and further trading to strategic investors at reasonably "correct" (much higher) prices.

A special attention should be paid to the negative value premiums given in (Tables 2,3). This tendency has already been reasoned hereinabove (footnote 3). The abovementioned can be added with the following marks:

- When switching from the value of one share of the stakes having less control to the value of one share of the stakes having more control, a negative value control premium is resulted in a positive value control discount.

- When switching from the value of one share of the stakes having less control to the value of one share of the stakes having more control, a negative value control premium occurs if a unit cost of control (value of one share of the stake), when switching to a larger stake, decreases because of the greater rise in the number of shares

Table 6: Calculation of the share of control depending on the entitlements of the stake being evaluated and equity capital structure.

#	The size of the stake (% of voting shares)	The sum of the points awarded to the corresponding stake	The structure of the equity capital, the sum of the points awarded to the remaining (except the one being estimated) consolidated stakes (10% and greater) and determination of the share of control accounting for the stake being evaluated
1	10÷25% -1	2	1. Consolidated shares of all other stakes are less than 10%: in this situation, it is difficult to draw a concrete conclusion; much depends on the activity of other shareholders, their ability and willingness to participate in the General Meeting of Shareholders. At the same time, it is clear that in such a situation, control over the proportional ownership share is observed. 2. A 12% stake is assessed, the remaining shares of 10%, 10%, 15%, 17%, 32% of the remaining stake. In this situation, the estimated share holding has 2 points, and the remaining packages have 2 + 2 + 2 + 9 = 15 points. Hence, the share of control of the estimated stake will be: 2 / (2 + 15) = 11.76%. 3. An 18% stake is assessed, with other packages exceeding 10% being 10%, 15%, 27%, 30%. In this situation, the estimated share holding has 2 points, and for the remaining stakes have 2 + 2 + 7 + 9 = 20 points. Hence, the share of control of the estimated stake will be: 2 / (2 + 20) = 9.09%. 4. A 11% stake is estimated, the shares of the remaining stake, from 10% and above, are: 10%, 27%, 52%. In this situation, the estimated share holding has 2 points, and the remaining stake have 2 + 7 + 44 = 53 points. Hence, the share of control of the estimated stake will be: 2 / (2 + 53) = 3.64%. 5. A 16% stake is estimated, the share of the remaining packages exceeding 10% is 51%. In this situation, the estimated share holding has 2 points, and the rest stakes have 44 points. Hence, the share of control of the estimated stake will be: 2 / (2 + 44) = 4.35%. 6. A 20% stake is estimated, the shares of the remaining stake exceeding 10% are: 75%. In this situation, the estimated share holding has 2 points, and the rest stakes have 72 points. Hence, the share of control of the estimated stake will be: 2 / (2 + 72) = 2.70%.
2	25	4	1. Consolidated shares of all other stakes are less than 10%. In such a situation, the share of control will significantly exceed the proportionate share of ownership, but it is unclear how much (additional analysis is needed). 2. A 25% stake is evaluated, the share of the remaining stake, from 10%, is 10%, 15%, 32%. In this situation, the estimated share holding has 4 points, and the remaining stakes have 2 + 2 + 9 = 13 points. Hence, the share of control of the estimated stake will be: 4 / (4 + 13) = 23.53%. 3. A 25% stake is assessed; the remaining shares of other stakes exceeding 10% are 27%, 32%. In this situation, the estimated share holding has 4 points, and the remaining stakes have 7 + 9 = 16 points. Hence, the share of control attributable to the estimated package will be: 4 / (4 + 16) = 20%. 4. A 25% stake is estimated, the shares of the remaining stake exceeding 10% are: 11%, 51%. In this situation, the estimated share holding has 4 points, and the remaining stakes have 2 + 44 = 46 points. Hence, the share of control of the evaluated stake will be: 4 / (4 + 46) = 8%. 5. A 25% stake is estimated, the share of the remaining stakes exceeding 10% is 51%. In this situation, the assessed shareholding has 4 points, and the rest of the stakes have 44 points. Hence, the share of control of the estimated stake will be: 4 / (4 + 44) = 8.33%. 6. A 25% stake is estimated, the shares of the remaining stake exceeding 10% are: 75%. In this situation, the estimated share holding has 4 points, and the rest of the stakes have 72 points. Hence, the share of control of the estimated stake will be: 4 / (4 + 72) = 5.26%.
3	25%+1÷ ÷30%- 1	7	1. Consolidated shares of all other stakes are less than 10%. In such a situation, the share of control will significantly exceed the proportionate share of ownership, but it is unclear how much (additional analysis is needed). 2. A 28% package is estimated, the share of the remaining stake, from 10%, is 10%, 15%, 32%. In this situation, the evaluated shareholding has 7 points, and the remaining stakes have 2 + 2 + 9 = 13 points. Hence, the share of control of the estimated stake will be: 7 / (7 + 13) = 35%. 3. A 27% stake is estimated, the share of the remaining stake exceeding 10% is 51%. In this situation, the estimated share holding has 7 points, and the other consolidated stake have 44 points. Hence, the share of control of the estimated stake will be: 7 / (7 + 44) = 13.73%.
4	30%÷ 50%	9	1. Consolidated shares of all other stakes are less than 10%. In such a situation, the share of control will significantly exceed the proportionate share of ownership, but it is unclear how much (additional analysis is needed). 2. A 30% stake is estimated, the rest of the stakes, from 10%, are: 10%, 15%, 32%. In this situation, the estimated share holding has 9 points, and the remaining stakes have 2 + 2 + 9 = 13 points. Hence, the share of control of the estimated stake will be: 9 / (9 + 13) = 40.91%. 3. A 37% satke is estimated, the shares of the remaining stakes, exceeding 10%, are: 12%, 51%. In this situation, the estimated stake has 9 points, and the remaining stakes have 2 + 44 = 46 points. Hence, the share of control of the estimated stake will be: 9 / (9 + 46) = 16.36%. 4. A 39% stake is evaluated, the shares of the remaining stakes exceeding 10% are: 12%, 26%. In this situation, the estimated stake has 9 points, and the remaining stakes have 2 + 7 = 9 points. Hence, the share of control of the estimated stake will be: 9 / (9 + 9) = 50%. 5. A 39% stake is estimated, the share of the remaining stakes exceeding 10% is 55%. In this situation, the estimated stake has 9 points, and the rest stakes have 44 points. Hence, the share of control of the estimated stake will be: 9 / (9 + 44) = 16.98%.

5	50%+1÷ ÷ 75%-1	42	1. Consolidated shares of all other stakes are less than 10%. In such a situation, the share of control will significantly exceed the proportionate share of ownership, but it is unclear how much (additional analysis is needed). 2. A 55% stake is evaluated, the remaining shares of other stakes exceeding 10% are: 12%, 26%. In this situation, the estimated share holding has 42 points, and the remaining stakes have 2 + 7 = 9 points. Hence, the share of control of the estimated stake will be: 42 / (42 + 9) = 82.35%. 3. A 51% stake is evaluated, the rest of the stakes, from 10%, are: 10%, 30%. In this situation, the evaluated shareholding has 42 points, and the remaining stakes have 2 + 9 = 11 points. Hence, the share of control of the evaluated stake will be: 42 / (42 + 11) = 79.25%.
6	75%÷ 100%	72	1. Consolidated shares of all other stakes are less than 10%. In such a situation, the share of control will be close to 100% (the minimum lower limit can be considered with a high degree of confidence: 1- 2 / (2 + 72) = 97.28%). 2. A 75% stake is assessed, the shares of the remaining stakes, exceeding 10%, are: 11%, 13%. In this situation, the estimated shareholding has 72 points, and the remaining stakes have 2 + 2 = 4 points. Hence, the share of control of the estimated stake will be: 72 / (72 + 4) = 94.74%. 3. A 75% stake is evaluated, the shares of the remaining stakes exceeding 10% are: 25%. In this situation, the estimated stake has 72 points, and the rest stakes have 4 points. Hence, the share of control of the estimated stake will be: 72 / (72 + 4) = 94.74%.

as compared with the increase in value because of the additional elements of control.

- Empirical observations prove the presence of negative value premiums.

To confirm the last of the statements given hereinabove, let us provide some valuation market data for 1st quarter 2017 from the quarterly CONTROL PREMIUM STUDY reports published by the FACTSET MERGERSTAT.

In order to obtain unbiased and accurate pricing information, the scope of this study has been narrowed to completed transactions where the target company was publicly traded. For domestic transactions, Premiums ranged from -31.0% for WaferGen Biosystems, Inc. to 104.3% for Media General, Inc.. For international transactions, Premiums ranged from -93.7% for Tesco PLC (Tesco Kipa Kitle Pazarlama Ticaret Lojistik ve Gida Sanayi AS) to 124.3% for Exxon Mobil Corp. (Mobil Oil Nigeria PLC). Mergerstat does not include negative premiums in calculating average and median statistics. If negative premiums had been included in the calculations for the 1st Quarter 2017, the results would have been as follows:

Source: FACTSET MERGERSTAT Global Mergers and Acquisitions Information. CONTROL PREMIUM STUDY, 1ST QUARTER 2017. P. ii.

As it is obvious, the premiums including the deals with negative value premiums in the left-hand side of the Table are lower than those excluding the deals with negative premiums in the right-hand side of the Table.

Summarizing the aforementioned, let us propose some more helpful formulas for calculating control premiums. The additional formula applicable to equation (55) (Table 3) for estimating a relative control premium, when switching from the proportional value of one share of a 100% stake to the value of one share of a nc% controlling stake, can be written as [14]:

$$\delta pr(p_{100} - p_{nc}) = \frac{cv.(a(\delta n_c) - \delta(n_c))}{\delta(n_c).MC_c} = c_d \times \left[\frac{a(\delta n_c)}{\delta(n_c)} - 1\right] = 0,167 \times \left[\frac{0,75}{0,51} - 1\right] = 0,078.$$

(38)

The amount of a control premium for the shares of a controlling stake (whose share equals $\delta(n_c)$) having 100% control ($a = 1$) in comparison with the value of one share of a non-specific

(proportional) stake in the absence of blocking stakes:

$$\delta pr(p_{100} - p_{nc}, a = 1) = \frac{pr_{100} \times (1 - \delta(n_c))}{(1 + pr_{100}) \times \delta(n_c)} = \frac{0,2 \times (1 - 0,51)}{(1 + 0,2) \times 0,51} = 0,160,$$

(39)

where pr100 – value of control premium with 100% powers of control.

As indicated above, for the condition of the example being discussed, the estimated value of one share in a non-specific (proportional) stake is 120/100 = 1.2. Accordingly, applying expression (33), it is possible to obtain an estimate of the value of one share of a 51% stake having 75% control: 1.2 × (1 + 0.078) = 1.294. If the holders of this stake had 100% control powers, the value of one share of their shareholding could be: 1.2 × (1 + 0.16) = 1.392 (the value of this premium (16%) can be obtained using expression (36)).

Evaluating the amount of control

As it follows from the equation given hereinabove, in order to implement estimates in determining control premiums and control discounts, it is necessary to determine the value of the amount of control (in the given formulas, parameter "a"). As it was previously pointed out, an accurate assessment of the controlling interest share (a) held by the owner of the stake being evaluated can be carried out through a rigorous analysis of the structure of the equity capital, taking into account the distribution of all legally significant consolidated stakes (with due regard to the powers provided to their holders by the current laws), existing alliances, as well as allocation of seats within the Board of Directors of the issuing company.

The description of the methods for determining the amount of control, presented in the evaluation literature, can be found, for example, in [3] and [4]. From the practical point of view, the second of the above papers (developed in subsequent publications of this author) is of particular interest. Its application is based on the rights of shareholders arising from the legislation on joint-stock companies (regarding shareholders' rights depending on the size of a share holding), and on determining the number of potential buyers of a stake from the owners of 1% or more stakes or third-party buyers. While retaining the general concept of this article (in terms of accounting for the different scope of powers depending on the size of the shareholding acquired), the author proposes an alternative possibility of determining the amount of control for the subsequent application of the obtained result (the value of the parameter "a") in

calculating the cost of various stakes and / or control premiums and control discounts. The idea of the proposed method for determining the amount of control is as follows: first, the rights of shareholders arising from the current legislation are to be analyzed (Table 4 and Order 208-FZ "On Joint-Stock Companies" [15]).

Source: Order 208-FZ "On Joint-Stock Companies" and comments by lawyers Nikolay Chernov and Pavel Kalinin, the lawyers of the Monastyrsky, Zyuba, Stepanov & Partners Bar Association.

Then each right is assigned a certain value in points (Table 5) [17], and the calculation of the sum of points attributable to the relevant package is carried out (it should be borne in mind that a larger package includes all the powers assigned to a smaller package).

Source: The result of the survey carried out by the author of the paper.

After that, depending on the assigned ("awarded") points and the structure of the share capital, the control interest share of the stake being evaluated is calculated: this share is determined as the quotient from the division of the points awarded to the stake by the total amount of the same points awarded to the remaining largest stakes (from 10% and more) (Table 6).

Source: Author's calculations

Having received the value of control rate, it should be later used when substituting parameter "a" in the calculation formulas given above (see sections 2-3).

As it has already been mentioned above (see reference 8) in a case when major shareholders may become the holders of strategically more valuable block when acquiring the minority block of shares, they may offer the minority shareholders a premium to the current market value, and sometimes such a premium that the value of one share in the minority or blocking parcel to be acquired may finally exceed the value of one share even in absolutely controlling block. To estimate the maximum premium which the holders of one block may pay to the holder of another block in this case, we shall introduce new parameters:

Na - a number of shares composing "the parcel-acquirer", Na =23,

Nt – a number of shares composing the target minority block, Nt = 3.

Let it be also known that control value (CV) equals 20, the value of one share (p) equals 1, and control rate (a(δnb)) accounting for the blocking parcel equals 25%.

Thus, the maximum premium when acquiring the target parcel will make up [18]:

$$PR_b = \frac{p.(N_b - N_a) + a(\delta n_b).CV}{N_t} - p = \frac{p.N_t + a(\delta n_b).CV}{N_t} - p = \frac{a(\delta n_b).CV}{N_t} =$$
$$\frac{1.(26-23)+0,25.20}{3}-1 = \frac{1.3+0,25.20}{3}-1 = \frac{0,25.20}{3} = 1,67, \quad (40)$$

where PRb – a maximum calculated premium paid out for transferring the parcel-acquirer to a blocking parcel ("b" index means belonging to a blocking parcel).

According to the results received in (37), the maximum value proposed for purchasing each share of this parcel may equal 1+1.67=2.67 – that is much higher than the value of one share composing the controlling block. In reality, of course, it is needless to expect that the owner of such minority parcel will be offered maximally possible premium to the current market value as its acquirers consider not least reselling the acquired strategic block in future. It is more likely that the extra payment will make up not more than 10…30% [19] of the maximum level, i.e. approximately 0.17…0.5 and then the value offered for the share will range 1.17…1.5.

The equation (37) may be used to calculate maximum premium rate in all cases of acquiring minimal possible block of shares followed by strategic switching. For example, if in previous case the major shareholders had the blocking parcel in disposal totaling Nb=47 shares (with the control rate of 25%) and wanted to acquire the parcel composed of 4 shares that would allow them to become the owners of the controlling parcel (with the control rate of 75%), the maximum premium rate for coming into the controlling block could be [20]:

$$PR_c = \frac{p.(N_c - N_b) + CV.(a\delta n_c) - a(a\delta n_b))}{N_t} - p = \frac{Cv.(a(\delta n_c) - a(\delta n_b)}{N_t} =$$
$$\frac{1.(51-47)+20.(0,75-0,25)}{4}-1 = \frac{20.(0,75-0,25)}{4} = 2,5. \quad (41)$$

In a case when the excess block of shares (for example when its seller does not wish to split the block), the value of excess shares does not exceed their market value as a part of the minority block, so the premium rate per share will be lower than that maximum rate which would be possible in a case when the minimum necessary number of shares. For example, if in the previous case the block composed of not 4 but 10 shares, the maximum premium rate would total 1.

Determining control value based on tender bids cost

Let us suppose that the evaluator is aware of one of the tender bids costs which was accepted: ptc — the cost of the accepted tender bid per share as a part of the controlling block when acquiring the whole block, or pth — the cost of the accepted tender bid per share as a part of the blocking parcel when acquiring it.

Besides, the evaluator is aware of the fact that there is the second strategic parcel in the share capital and at the same time he is not aware of the cost of 100% block of shares with due regard to control rate value (parameter MCc). Then determining the other unknown cost variable (ptb, if ptc is known, or ptc if ptb is known) is possible only if there are data on the parcel size, distribution of the control between them and the value of the shares from minority blocks.

Determining control value in this case is not obligatory (but obligatory for people taking part in a deal), but in a greater degree auxiliary procedure so that it became possible to apply the formulas stated above with the purpose to estimate one of the unknown value.

To determine the control value based on the bids costs we shall take the equation of the values of strategic blocks identified by the formula (25) and (21) and the costs of tender bids of these blocks.

If the cost of the accepted tender bid for the controlling block is known:

$$P_c = p_{tc} \cdot N_c, \quad (42)$$

then equaling this equation to previously received one for the value of the controlling block (see (25)) we shall have the control value:

$$CV = \frac{(p_{tc} - p).N_c}{a(\delta n_c)}. \qquad (43)$$

If the cost of the excepted tender bid for the blocking parcel is known:

$$p_b = p_{tb}.N_b, \qquad (44)$$

then equaling its meaning to the previously obtained calculated value of the blocking parcel if there is controlling block (see (21)) we shall receive another equation for the control value:

$$CV = \frac{(p_{tb} - p).N_b}{a(\delta n_b)}. \qquad (45)$$

By using equations (40) and (42) we can receive the formula to determine theoretical (calculated) value per share from the blocking parcel ptb at the known value of one share ptc in the accepted tender bid on purchasing the controlling block:

$$p_{tb} = p + \frac{p_{tc} - p).a(\delta n_b)}{N_b.a(\delta n_c)}, \qquad (46)$$

or the equation for ptc at the known value of pth:

$$p_{tc} = p + \frac{N_b.(pt_b - p).a(\delta n_c)}{N_c.a(\delta n_b)}. \qquad (47)$$

It should be noted that the tender bids costs, as a rule, do not equal estimated market values of corresponding strategic blocks as in reality such blocks may have only investment value which is determined in this or that degree by the tender bids cost. Possible difference of investment value from estimated market value in this case are conditioned first of all by two factors: firstly, by the ratio between the necessary number of shares and number of shares floating freely on the market; secondly, by the unawareness of the people taking part in a deal of the proper calculation methodology.

Conclusions

Summing up the article it must be noted

Premiums for the control presence or increase as well as the discounts for the control lack or decrease are familiar actions leading to the differences in the value of the shares from various blocks in comparison with the Estimation of their proportional value.

Estimation of their proportional value

The paper presented above proposes the calculation methodology of premiums and discounts [21] based on that market members or qualified evaluators register the differences in the unit value of the share from various blocks, the share capital structure as well as various powers envisaged by applicable law on joint-stock companies in relation to shareholders. This methodology may be applied only in the cases when people estimating the shares have access to the information on the differences in the unit values of the shares from various blocks: if these data are available, it is likely that more substantiated premium and discounts estimates will be received (as well as the estimates of the relevant blocks values) if compared to the empirical data; if these data are not available it remains possible to apply the data of empirical observations over the results of the deals given, in particular, in the control premium study reference book published by fact set merger stat.

CONTROL PREMIUM STUDY reference book published by FACTSET MERGERSTAT

The analysis of the data presented in this article allows us to conclude that when switching from one to another control level (while considering two various blocks of shares) the varying share unit value is influenced divergently by the change in the powers granted to the shareholders and in the number of shares from the block. For example, if when switching to a larger block the powers/ share of control is increased by 10%, and the overall number in the block – by more than 10%, the value of one share from a larger block may decrease that amounts to the discount (negative premium) for the control.

When applying the methodology proposed in this article it is necessary to take in consideration the following. If not less than two approaches were used during the estimation, one of which is comparative (based on the deals with minority blocks), it is allowed to average only intermediate calculations obtained on the basis of comparative approach (for example, in the case when several multipliers were used) or the calculations of income or cost approaches. In all these cases no discounts or extra payments can be applied to intermediate calculations. Also, it is not allowed to average the result of comparative approach (obtained on the basis of the deals with minority blocks) with the result of income and/or cost approaches.

Applying the proposed methodology will make it possible to calculate discounts/premiums for the absence/lack of control more reasonably and avoid (basically) voluntarism which happened to exist in this issue, as:

1. Correct logical and economical principles formed the basis of calculations;

2. Control value estimate relies on market value of this figure;

3. Control rate estimate considers the share capital structure and is based on applicable laws on joint-stock companies.

4. Correct mathematical calculations are used.

Concluding studying this issue the author of this article considers it is necessary to make several notes in terms of how the control values was regarded by A. Damodaran [5], [6]. The essence of Damodaran`s approach can be briefly expressed in the following formulas:

$$MC_{opt} = MC + p_R.(MC_{opt} - MC), \qquad (48)$$

$$MC_{opt} = MC + p_R.(MC_{opt} - MC), \qquad (49)$$

where, CV – control value,

MCopt – the value of the company with an optimal organizational structure, MC – the value of the company, the value of the company with current management,

PR – the possibility of the takeover of the company by third-party investors and/or of the beginning of a successful restructuring activity.

According to Damodaran`s approach, the control value is reflected on market capitalization of the company only when there is a possibility that its value increases. Such possibility is

determined by the vision of potential optimally restructured value and by the feasibility of the plan on restructuring activity in practice. Considering such a possibility (possibilities) is undoubtedly a benefit of Damodaran`s approach.

At the same time the application of Damodaran`s approach to estimate the premium for control appears to be slightly problematic. On the one hand, Damodaran`s approach does not take into account varying powers of different shareholders. On the other hand, when implementing Damodaran`s approach to premiums identification we come across with some technical complications: firstly, it is rather time-taking and speculative due to the necessity to estimate optimally restructured value; secondly, there appear uncertainties with referring the restructuring plan implementation to as a successful one. And, finally, thirdly, this approach restricts the comparative approach usage, namely those methods which are based on the data on minority blocks quotations. This restriction is conditioned by internal cyclicity: the estimate of final share capital value based on different approaches results is possible only weighting homogeneous quantities – with or without control premiums; however, the control value itself at this stage is unknown so far. Besides, it should be mentioned that Damodaran`s approach does not account for "grabbing the biggest piece of pie" by controlling shareholders.

According to the approach presented in this article, control value is determined by the market itself, by using the current state (i.e. it is unnecessary to estimate optimally restructured value and possible control switching to more effective shareholders-executives and successful restructuring plan).

At the same time, Damodaran`s control provoked the following thoughts. Estimating the control value by the formula (1) implicitly implies receiving in most cases positive results. To achieve this, the income and cost approaches results must exceed the results of the comparative approach obtained on the basis of minority blocks quotations in the company-analogue. But we should not exclude the possibility that estimating by formula (1) we can receive negative results. What might it mean? Would not it mean a value loss due to ineffective company management? If so, then this is the case when we could try using Damodaran`a approach when estimating premium value. However, the described case [the excess of the company value estimated at the minority level over the value estimated at the controlling value] may also be interpreted in other way. As it has been noted before, the block size increase accompanied by control rate enhancement results in decreasing the block liquidity degree as major block of shares require longer time period and efforts for their sale. Another possible interpretation may imply "overheated market" and approaching market trends change. And finally, it may also mean inaccuracies made by the evaluators when performing estimating calculation.

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