# Adjusted Survival Tree Models for Genetic Association: Prognostic and Predictive Effects 

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#### Abstract

A general method to create adjusted survival trees is developed. Prognostic survival trees have been used to automatically uncover complicated GxG and GXE interactions, however scientist soften want to uncover this structure while adjusting for confounding factors not of interest. Interaction survival trees automatically identify the best treatment choice for patients and area promising model to enable personalized medicine, but simulations to assess their performance on the high dimensional data found in personalized medicine have not been conducted. We develop a general framework to adjust for confounding factors in prognostic and interaction survival trees. These factors are numerous in practice and can include age, gender, study site in a randomized multicenter clinical trial, and the principal components of ancestry difference to control for population stratification in genetic studies. Extensive simulations show the performance of our methods to be well controlled under the null and are robust to large dimensional covariate spaces under the alternative. In a real data example, our adjusted interaction tree successfully identifies subgroups of head and neck cancer patients that respond positively to having antioxidant vitamins added to their treatment regime. Applications, guidelines for use, and areas for future research are discussed.


Keywords: Pruning; Splitting; MLE; Tree models

## Introduction

Tree-based methods were first introduced by Morgan and Sonquist [1] and greatly extended by Breiman, Freidman, Olshen, and Stone [2]. The flexibility of tree-based methods are appealing as they can automatically detect complicated interactions, naturally handle missing data via surrogate splits [2], select covariates in the presence of high dimensional data, and can be easily extended by ensemble methods to create random forests [3]. Early work on the development of survival trees began shortly after the original classification and regression tree methods and is summarized by LeBlanc and Crowley [4]. Recent research has extended survival trees to more complicated situations, such as multivariate data [5], extended ensemble methods to survival trees such as in random survival forests [6], and introduced new splitting rules that partition the covariates pace based on interaction with a specified covariate [7].

Tree-based methods are particularly appealing for scientific studies as they automatically partition in the covariate space in a way that mimics a human's natural decision making process. Indeed, survival trees have often been used to uncover complicated GxE and GxG interactions and classify the prognosis of patients [8]. However in these genetic studies clinician softens want to uncover the effects of a set of genetic and environmental factors of scientific interest while adjusting for the effect of confounders which are not of direct interest. Although such adjusted trees have been developed for continuous and binary out comes [9] they have not been developed for the time-to-event outcomes of ten found in cancer research.

Recently personalized medicine, which is the idea of giving the right treatment to the right person based on their genetic, clinical
and demographic characteristics has become of increasing interest in the clinical community [10-13]. Developing statistical methods to help enable personalized medicine in cancer research has many challenges. They should be able to handle high dimensional genetic data, focus on prediction of best treatment rather than prognosis, adjust for clinical confounders, work on survival data, and be easily interpretable for clinical decision making. Common approaches to identify subgroups of patients who respond differently to treatment include sub group analysis, which is not statistically sound [14], and regression modeling, which cannot automatically identify complex subgroups [15]. Survival trees have been modified to partition the covariate space based on differences in response to treatment [7], however, they cannot adjust for confounding and simulations have not been done to assess their effectiveness on large scale genetic data found in personalized medicine.

In this article we develop a general framework to create survival trees that partition the covariates pace based on the effects of asset of genetic and environmental factors that are of scientific interest while adjusting for possible confounders which are not. Such confounders are numerous in practice and can include age, gender, study site in a multicenter randomized clinical trial, and the principal components of ancestry difference to control for population stratification. We apply this framework to prognostic and interaction survival trees. Here, prognostic survival trees are tree-based methods which generate rules to classify the prognosis of patients by partitioning them based on combinations of covariates; while interaction survival trees generate rules to classify the best treatment choice of patients by partitioning them based on combinations of covariates which have strong interaction with treatment.

Next we will introduce the basic structure of a recursive partitioning algorithm used to create trees. We then define our new splitting rules, pruning algorithm, and a method to choose the final tree. Extensive simulations are the presented to evaluate the performance of our innovative methods under several scenarios, including the high dimensional covariate space commonly found in personalized medicine. An application of our methods to a randomized clinical is performed, showing its significant clinical relevance. We then close with a discussion on the practical use and implementation of our methods, as well as on areas of further research.

## Methods

## Algorithm overview

Tree based models are created using a recursive partitioning algorithm. These algorithms usually consist of three parts: a splitting rule, a pruning algorithm, and a method for selecting the final tree. The splitting rule partitions the covariates pace $\chi$ into many groups. It is applied recursively until there are very few observations in each group, or a pre-specified maximal number of groups are created [2]. This partition can be represented as a tree $T$, with terminal nodes $|\tilde{T}|$ corresponding to the partition of the covariate space $\chi$ into $|\tilde{T}|$ subsets. This large tree usually over fits the data and will perform poorly out-of-sample. Thus a sub tree is chosen as the final tree. The space of all possible sub trees is large, and a pruning algorithm is used to efficiently search this space and find the optimal sub trees. The final sub tree is then selected either using a test set or a resampling technique [16].

## Splitting

For simplicity, in this paper, we only consider the case of making binary splits to single covariates. A potential splits of a covariate $c$ can then be characterized as follows. If $c$ is binary, then $s$ is the trivial partition of $c$. If $c$ is continuous or ordinal, $s$ can be any binary partition of $c$ such that all elements in one partition are less than those in the other. If $c$ is categorical then $s$ can be any binary partition of the levels of $c$.

To partition a node $h$, find the splits such that some measure of improvement $G(s, h)$ is maximized.

$$
G=\left(s^{*}, h\right)=\max _{s \in S_{h}} G(s, h)
$$

where $S_{h}$ is the set of all binary splits that can be made at node $h$. If there is more than one terminal node to partition, then find the best splits * for each $h \in H$ and split the node with the maximal improvement. In the case of ties randomly select one of the splits with maximal improvement.

Recall the standard survival data set-up with data for observation $i$ of the form $\left(y_{i}, x_{i}, \delta_{i}\right)$ where $x_{i}$ is the covariate vector for observation $i$ and survival time $y_{i}$ is censored if $\delta_{i}=0$ and an event if $\delta_{i}=1$. The Cox model assumes that

$$
\lambda(t \mid X)=\lambda_{0}(t) e^{\beta^{\prime} X}
$$

where $x$ are covariates, $\lambda(t \mid x)$ is the hazard at time $t$ given $x$ and $\lambda_{0}$ is some baseline hazard function. The maximum likelihood estimate (MLE), $\hat{\beta}$, of the parameters $\beta$ is found without specifying $\lambda_{0}$ by maximizing the log-partial- likelihood

$$
\ell(\beta)=\sum_{i=1}^{K} \delta_{i}\left(\beta^{\prime} X_{i}-\log \sum_{j \in R\left(t_{i}\right)} \exp \left(\beta^{\prime} X_{j}\right)\right.
$$

with respect to $\beta$ where $t_{i}$ is the survival time for observation $i$ and $R\left(t_{i}\right)$ is the set of observations $i$ that are at risk at time $t_{i}$.

Consider two Cox models, $m_{0}: \log (\lambda)=\beta^{\prime} x_{0}$ and $m_{1}: \log (\lambda)=\beta^{\prime} x_{0}$ $+\beta_{1}^{\prime} x_{1}$ is said to be nested in $m_{1}$ and the Likelihood Ratio Test (LRT)

Statistic corresponding to the hypothesis test $H_{0}: \beta_{1}=0$ is

$$
2\left(\ell_{m 1}\left(\hat{\beta}_{m 1}\right)-\ell_{m 0}\left(\hat{\beta}_{m 0}\right)\right) \stackrel{H_{0}, n \rightarrow \infty}{\sim} \chi_{\operatorname{rank}(X 1)}^{2}
$$

we define two new splitting rules which can be used to create adjusted prognostic trees and adjusted interaction trees respectively.

Definition1. $G_{a}(s, h)$ is the LRT statistic corresponding to $H_{0}: \beta_{s}=0$ in the Cox model $\log (\lambda)=\beta_{c} x_{c}+\beta_{s} x_{s}$

Definition 2. $G_{a i}(s, h)$ is the LRT statistic corresponding to $H_{0}: \beta_{t s}=0$ in the Cox model $\log (\lambda)=\beta_{c} x_{c}+\beta_{t} x_{t}+\beta_{s} x_{s}+\beta_{t s}\left(x_{t} \times x_{s}\right)$

Also recall the non-adjusted interaction survival tree split $G_{i}(s, h)$ defined in [7] as the LRT statistic corresponding to $H_{0}: \beta_{t s}=0$ in the Cox model $\log (\lambda)=\beta_{t} x_{t}+\beta_{s} x_{s}+\beta_{t s}\left(x_{t} \times x_{s}\right)$; Here $\mathrm{x}_{\mathrm{c}}$ is a vector of confounding variables, $x_{s}$ is an indicator of the potential splits, which is a binary partition of some covariate $c, x_{t}$ is a treatment with $\geq 2$ levels, and $x_{t} \times x_{s}$ is the interactive term of the treatment and the splits. $\beta_{s}$ is the effect of the splits, $\beta_{t}$ is the effect of the treatment, $\beta_{c}$ is the effect of the confounders, and $\beta_{t s}$ is the effect of the interaction of the treatment and the splits. $G_{a}(s, h)$ is the split for the adjusted survival tree and $G_{a i}(s, h)$ is the split for the adjusted interaction survival tree. The best split can be interpreted as the one that creates the two child nodes with the most statistically significant adjusted difference in prognosis and response to treatment respectively.

## Pruning

The split-complexity $G_{\alpha}(T)$ can be defined as

$$
G \alpha(T)=G(T)-\alpha|S|
$$

where $S$ is the set of internal nodes of tree $T,|S|$ is the cardinality of $S, \alpha \geq 0$ is the complexity parameter, and $G(T)$, the goodness of split of tree $T$, is the sum of the split improvement statistics over the tree.

$$
G(T)=\sum_{h \in S} G(h)
$$

Consider all possible sub trees of a large tree $T_{0}$, although this space is large it is easy to see that when $\alpha=0$ the entire tree $T_{0}$ will have the largest split complexity, and when $\alpha$ is sufficiently large the null tree with no splits $T_{m}$ will. Leblanc and Crowley [16] extend this argument and define a pruning algorithm based on splitcomplexity that efficiently finds the sub trees $T_{m}<\ldots<T_{k}<\ldots<T_{0}$ and corresponding complexity parameters $\infty>\alpha_{m}>\ldots>\alpha_{k}>\ldots>\alpha_{1}>\alpha_{0}=0$ such that $T_{k}$ has the largest goodness-of-split of any sub tree for all $\alpha_{k} \leq \alpha<\alpha_{k+1}$. They prove the theoretical properties of this algorithm directly, and as a special case of CART [2]. After building a large tree with one of our new splitting rules $G_{a}$ or $G_{a i}$ we efficiently find the optimal sub trees by using this algorithm directly, letting $G$ be the new splitting rule used to build the original tree.

## Selection of the final tree

After finding the optimally pruned sub trees with the above algorithm, we may still wish to choose a final tree. Since the splits


Figure 1: Simulated Tree Structures. The following tree structures were simulated to test our adjusted prognostic and adjusted interaction tree algorithms.
used to make a tree are adaptively chosen as the maximum of several potentially correlated LRT statistics, the split complexity $G \alpha_{c}(T)$ is larger than would be expected if the splits were chosen a-priori. If we have a large sample we can get an 'honest' estimate of $G \alpha_{c}$ ( $T$ ) by using the following method.

First split the data into a training set and test set. Next build a large tree with the training set and find the optimal sub trees with the algorithm in the above section. Finally force the test set down each of the sub trees. The final tree is the one that maximizes $G \alpha_{c}(T)$ where $G \alpha_{c}(T)$ is calculated using the test set. We recommend using $\alpha_{c}=4$. This roughly corresponds to the 0.05 significance level of the split [4].

When the data cannot be split into training and test set we propose choosing the final tree with a 5 -fold cross validation based method. First build a large tree with the full data and find the optimal sub trees using the algorithm from section 2.3. Toper form the 5 -fold cross validation first partition the observations into 5folds $L_{i, j}=(1, \ldots, 5)$ and build 5 trees $T_{(-j)}$ on samples $L_{(-j)}$. For each $\alpha_{k}$ and $T_{(-j)}$ find the optimal sub tree $T_{(-j)}, k$ and force $L_{j}$ on $T_{(-j)}, k$ obtaining trees $T_{j}, k$. For each $T_{j}$, $k$ calculates the goodness of split $G\left(T_{i, k}\right)$ and take the mean over the folds to get $G(T ., k)$. Find $k^{*}=\max k G \alpha_{c}(T ., k)$ and if $G \alpha_{c}\left(T,, k^{*}\right)>0$ the final tree is $T k^{*}$, otherwise it is the null tree with no nodes.

## Simulation

For our two new splitting methods, $G_{a}(s, h)$ and $G_{a i}(s, h)$, we simulated our recursive partitioning algorithm under the true tree structure corresponding to the 'null hypothesis' of no splits, with one split, and with splits at position one and two. These tree structures can be seen in Figure 1. Failure times were simulated from the exponential distribution and censoring times from a uniform $(0, \gamma)$ distribution. $\gamma$ was chosen to have approximately $20 \%$ censoring. Three set $s$ of covariates, $x_{c}, x_{t}$ and $\mathrm{x}_{s}$ were generated. $x_{c}$ was a set of four potential confounders, two of which were truly associated with the outcome. $x_{t}$ was a balanced binary treatment. By default $x_{s}$ was 100 binary variables used to build the tree. All unassociated $x_{s}$ were assigned a random proportion, while associated $x_{s}$ had proportion 0.5 . Under the simulations of the null tree and tree with one split n was 500 , and under the simulation with two splits it was 1000 . For the prognostic tree the effect size of the true split $e_{s}^{\beta}$ was set to 2 and for the interaction tree the effect size $\mathcal{e}_{\text {ts }}^{\beta}$ of the interactive term of true split and treatment was set to 3.5 . When simulating two associated nodes, the properties of the first node were fixed while the second node varied. We also ran simulations of the non-adjusted splitting

Table 1: Tree performance under the null hypothesis given different sample size.

| Splitting Rule | Sample Size(n) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1000 | 900 | 800 | 700 | 600 | 400 | 300 | 200 |  |
| Ga | .029 | .045 | .051 | .048 | .061 | .049 | .064 | .054 |  |
| Gi | .049 | .068 | .054 | .042 | .047 | .038 | .032 | .014 |  |
| Gai | .071 | .070 | .051 | .069 | .059 | .047 | .045 | .022 |  |

Table 2: Tree performance under the null hypothesis given different number of potential splits.

| Splitting Rule | Number of Potential Splits |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1000 | 500 | 250 | 100 | 50 | 10 |  |
| Ga | .084 | .057 | .055 | .038 | .032 | .024 |  |
| Gi | .044 | .054 | .051 | .048 | .047 | .035 |  |
| Gai | .039 | .053 | .058 | .040 | .058 | .058 |  |

rule $G_{i}(s, h)$ on data with no confounders and compared the results with $G_{a i}(s, h)$. Under each setting large trees of size 10 were built, the optimal sub trees were found, and a final tree was selected with 1000 replications.

Extended simulations with $40 \%$ censoring, $60 \%$ censoring, 2:1 unbalanced treatment, weak correlation between the associated SNP and the confounder, and strong correlation between the associated SNP and confounder were performed. For each of these simulations the default parameter values defined above were used.

## Results

The model performance under the 'null' is shown in Table 1 and Table 2. The model performs well with only a $1.4 \%$ to $8.4 \%$ chance of selecting the wrong tree, and with no obvious trends when varying any parameters. Note that although in the selection of the final tree we penalize each split by 4 , which roughly correspond to the 0.05 confidence level for a $\chi^{2}$ random variable, this does not imply an expected $5 \%$ chance of selecting the wrong tree under the null.

Under the 'alternative' we first consider the probability that the true tree is identified as an optimally pruned sub tree by the pruning algorithm in section 2.3. If this does not occur, then our method to


Figure 2: Simulation Results: Adjusted Prognostic Tree, Two True Splits. The probability of our adjusted prognostic tree algorithm selecting the true tree as the final tree.


Figure 3: Simulation Results: Adjusted Prognostic Tree, One True Split. The probability of our adjusted prognostic tree algorithm selecting the true tree as the final tree.
choose the final tree out lined in section 2.4 will automatically fail. Of the times that the true tree is identified as an optimal sub tree, we then calculate the probability that the final tree we select with our resampling method is the correct one. Recall that the structure of the true tree is shown in Figurel.

For the prognostic tree, Figure 2 shows the probability of selecting the true tree when it has two splits and Figure 3 shows the power when the true tree has one split. Figure 4 and 5 show the probability of identifying the true tree as an optimal sub tree when the true tree has two and one splits respectively. Figures $6,7,8$, and 9 show the results for the interaction tree. The effect of modifying our simulation parameters is very similar across all splits, probabilities, and true tree structures.

In general the power to identify and select the correct tree increase as $n$ (and the underlying number of events) increases, the effect size increases, and the split becomes more balanced. Although the power decrease as the number of potential splits increases, this decrease is


Figure 4: Simulation Results: Adjusted Prognostic Tree, Two True Splits. The probability of our adjusted prognostic tree algorithm selecting the true tree as an optimal sub tree.


Figure 5: Simulation Results: Adjusted Prognostic Tree, One True Split. The probability of our adjusted prognostic tree algorithm selecting the true tree as an optimal sub tree.
relatively small, and it is quite robust to the high dimension of the covariate space. The power to identify the true tree as an optimal sub tree is usually quite high. See Figure 4, 5, 8 and 9 for more detail.

Table 3 and 4 show that the power to choose the correct tree decreases when the censoring increases to $40 \%$ and $60 \%$. Unbalanced 2:1 treatment, weak correlation between the associated SNP and the confounder, and strong correlation between the associated SNP and the confounder have relatively little impact on the power to choose the correct tree. The power to identify the correct tree as an optimal sub tree remains high even with $40 \%$ and $60 \%$ censoring.

## Prognostic tree results

When the true tree has two splits the power to select the correct tree decreases sharply when $n<750$, going from $90 \%$ at 750 down to $68 \%$ at 500 , and $26 \%$ at
250. The power stays around $94 \%$ from 1000 to 1500 . Modifying


Figure 6: Simulation Results: Adjusted Interaction Tree, Two True Splits. The probability of our adjusted iteration tree algorithm selecting the true tree as the final tree.


Figure 7: Simulation Results: Adjusted Interaction Tree, One True Split. The probability of our adjusted iteration tree algorithm selecting the true tree as the final tree.


Figure 8: Simulation Results: Adjusted Interaction Tree, Two True Splits. The probability of our adjusted interaction tree algorithm selecting the true tree as an optimal sub tree.
the balance of the split has a similar effect, with the power decreasing sharply when the balance is less than $1: 4$, going from $85 \%$ at $1: 4$, down to $69 \%$ at $1: 57,47 \%$ at $1: 9$ and $27 \%$ at $1: 19$. As the split becomes more balanced than 1:4 the power increases steadily until it reaches $93 \%$ when balanced. The power increases linearly from $24 \%$ to $88 \%$ as the effect size has increased from 1.3 to 1.8 . The power goes to $92 \%$ at 1.9 and reaches $94 \%$ at 2.25 . The power is quite robust to the number of potential splits, going from $92 \%$ at 10 potential splits, to $90 \%$ at 100 , $87 \%$ at 500 , and $86 \%$ at 1000 .

When the true tree has one split the power to select the correct tree decreases sharply when $n<400$, going from $91 \%$ at 400 down to $91 \%$ at 300 , and $52 \%$ at 400 . The power increases to $9 \%$ at 500 and stays relatively flat until 1000 . Varying the balance of the split has a similar effect. The power decreases sharply when the balance is less than $1: 3$ going from $92 \%$ at $1: 3$ down to $83 \%$ at $1: 4,70 \%$ at $1: 57,50 \%$ at $1: 9$ and $29 \%$ at $1: 19$. The power reaches $94 \%$ at $1: 1.86$, and reaches $95 \%$ when balanced. The power increases linearly from $16 \%$ to $81 \%$


Figure 9: Simulation Results: Adjusted Interaction Tree, One True Split. The probability of our adjusted interaction tree algorithm selecting the true tree as an optimal sub tree.
as the effect size is increased from 1.2 to 1.7. The power goes to $87 \%$ at $1.8,93 \%$ at 1.9 and $95 \%$ at 2 . The power is extremely robust to the number of potential splits, staying around $94 \%$ from 10 to1000.

## Interaction tree results

When the true tree has two splits the power to select the correct tree decreases sharply when $n<1000$, going from $90 \%$ at 1000 down to $78 \%$ at $750,50 \%$ at 500 , and $11 \%$ at 250 . The power increases to $94 \%$ at 1250 and reaches $95 \%$ at 1500 . Modifying the balance of the split has a similar effect, with the power decreasing sharply when the balance is less than $1: 4$, going from $80 \%$ at $1: 4$, down to $69 \%$ at $1: 57,38 \%$ at $1: 9$ and $12 \%$ at $1: 19$. As the split becomes more balanced than $1: 4$ the power increases steadily until it reaches $89 \%$ at $1: 1.5$, and finally reaches $90 \%$ at $1: 1$. The power increases linearly from $34 \%$ to $71 \%$ as the effect size is increased from 2 to 2.75 . The power goes to $79 \%$ at 3 , $90 \%$ at 3.5 and $95 \%$ at 4 . The power is extremely robust to the number of potential splits, staying around $92 \%$ from 10 to 1000 .

When the true tree has one split the power to select the correct tree decreases sharply when $n<500$, going from $92 \%$ at 500 down to $83 \%$ at $400,69 \%$ at 300 , and $40 \%$ at 200 . The power increases to $96 \%$ at 600 stays fairly stable, increasing to $97 \%$ at 1000. Varying the balance of the split has a similar effect. The power decreases sharply when the balance is less than $1: 4$, going from $78 \%$ at $1: 4$, down to $61 \%$ at $1: 57$, $40 \%$ at $1: 9$ and $16 \%$ at 1:19. Increasing the balance past $1: 4$ raises the power steadily until it reaches $92 \%$ at 1:2.5. The power is then stable until it is balanced. The power increases linearly from $37 \%$ to $76 \%$ as the effect size is increased from 2 to 2.75 . The power goes to $83 \%$ at 3 , $92 \%$ at 3.5 and $96 \%$ at 4 . The power is quite robust to the number of potential splits, going from $94 \%$ at 10 potential splits, to $92 \%$ at 100 and $89 \%$ at 1000 .

## Application to randomized clinical trial

Head and neck Cancer (HNC) is the $5^{\text {th }}$ most common type of cancer world-wide, with 650,000 new cases per year [17]. Most patients present with locally-advanced disease, with 5 year Overall Survival (OS) rates of about $50 \%$, which have not improved over the decades [18]. The use of antioxidant vitamins to supplement chemo and

Table 3: Tree performance under extended simulation settings. For each of the three simulated splitting rules Gai, Gi, and Ga and with true tree structures of 1 and 2 splits, the probability that the pruning algorithm picks the true tree as an optimal sub tree is shown. The result from the standard simulation is compared with simulations with $60 \%$ censoring, $40 \%$ censoring, $2: 1$ unbalanced treatment strong correlation between the associated SNP and the confounder and weak correlation between the associated SNP and the confounder. Extended simulation settings not relevant to a particular splitting rule are omitted.

| G | \# Splits | Standard | $60 \%$ <br> Censor | $40 \%$ <br> Censor | $2: 1$ <br> treat | Strong <br> Corr | Weak <br> Corr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gai | 2 | 0.90 | 0.43 | 0.75 | 0.84 | 0.89 | 0.92 |
| Gai | 1 | 0.92 | 0.56 | 0.84 | 0.89 | 0.92 | 0.93 |
| Gi | 2 | 0.92 | 0.46 | 0.75 | 0.84 |  |  |
| Gi | 1 | 0.96 | 0.56 | 0.83 | 0.90 |  |  |
| Ga | 2 | 0.93 | 0.56 | 0.83 |  | 0.87 | 0.94 |
| Ga | 1 | 0.95 | 0.73 | 0.91 |  | 0.90 | 0.94 |

Table 4: Tree performance under extended simulation settings. For each of the three simulated splitting rules Gai, Gi, and Ga and with true tree structures of 1 and 2 splits, the probability that the pruning algorithm picks the true tree as the final sub tree is shown. The result from the standard simulation is compared with simulations with $60 \%$ censoring, $40 \%$ censoring, $2: 1$ unbalanced treatment, strong correlation between the associated SNP and the confounder and weak correlation between the associated SNP and the confounder. Extended simulation settings not relevant to a particular splitting rule are omitted.

| G | \# Splits | Standard | $60 \%$ <br> Censor | $40 \%$ <br> Censor | $2: 1$ <br> treat | Strong <br> Corr | Weak <br> Corr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gai | 2 | 0.993 | 0.705 | 0.971 | 0.988 | 0.996 | 0.990 |
| Gai | 1 | 1.000 | 0.895 | 0.988 | 0.997 | 0.998 | 1.000 |
| Gi | 2 | 0.990 | 0.728 | 0.970 | 0.982 |  |  |
| Gi | 1 | 1.000 | 0.932 | 0.987 | 0.999 |  |  |
| Ga | 2 | 0.999 | 0.869 | 0.982 |  | 0.996 | 0.999 |
| Ga | 1 | 1.000 | 0.966 | 0.998 |  | 0.996 | 1.000 |

radiation therapy in cancer patients has had conflicting results. Some studies have shown that this regime leads to better survival outcomes, while others have shown that it leads to worse [19]. It is possible that this conflicting evidence could be driven by complex gene-treatment interactions. To explore such genetic hypotheses one must take care to control for population stratification, which is a major source of spurious results in genetic studies defined as the difference in allele frequencies in patients stemming from their ancestral differences [20]. The standard way to control for population stratification is to adjust for the main principal components of the ancestral differences [21]. Therefore we will use our adjusted interaction survival tree method to find subgroups of patients that respond most differently to antioxidant vitamins based on their genetic signature, while adjusting for the main principal components of their ancestral differences.

We analyzed the recurrence free survival of 540 patients with HNC from a randomized $\alpha$-tocopherol $/ \beta$-carotene placebocontrolled Genome Wide Association Study (GWAS) with 620,901 Single-Nucleotide Polymorphism (SNPs) [22]. After standard genetic quality control procedures, 515 patients, 261 randomized to the treatment arm and 254 randomized to a placebo provided genotype information for 543,873 SNPs. We calculated the principal components of the ancestral differences of the patients, and adjusted our tree by the top three. We built a tree with 10 splits from the 100 most prognostically significant SNPs. The dominant model was used for each SNP, transforming the SNP variable to an indicator of having


Figure 10: Real data adjusted interaction tree. The final tree selected by our pruning algorithm. The hazard ratio of treatment for each subgroup is written inside each node. The name of the SNP being split is written beside each node. The labels1 and 0 are an indicator of having at least one copy of them in oral lele in the respective SNP.
at least one minor allele. We pruned the tree with 1,000 boot strap samples and penalized each split by two. The final tree can be seen in Figure 10. Inside each node is the hazard ratio of treatment. A hazard ratio greater than 1 indicates that taking the antioxidant vitamin leads to a worse outcome than taking the placebo, while a hazard ratio less than 1 indicates that taking the vitamin leads to a better outcome. Although this is the best tree according to the automated pruning process, the tree can be further pruned by domain experts to remove statistically significant but clinically insignificant splits. Our method can identify subgroups of patients with specific genetic signatures for which the treatment has high efficacy.

## Discussion and Conclusion

The novel methods we have introduced can help translational research in genetic studies and personalized medicine. Scientists can use our methods to control for confounders when identifying complex GxG and GxE interactions or identifying the best treatment choice for patients based on their genetic profile. Moreover we have shown that the interaction survival tree can perform well with the large number of genetic factors often found in personalized medicine research. Once a tree is created and subgroups are identified, summary statistics such as hazard ratios of treatment, Kaplan-Meier curves [23], and median survival times for each group can be presented to clinicians. They can then use $t$ statistics, in combination with their clinical consideration to classify the prognosis or select the best treatment to their patients while controlling for potential confounders. Simulations have shown that the probability of selecting the wrong tree under the null hypothesis has been well controlled (at only $1.4-8.4 \%$ ) and that the power of selecting the true tree under the alternative hypothesis is usually high. To have adequate power there should be a sufficiently large number of events and interactive effect between the split and treatment. One must be particularly aware of the balance of the potential splits as power decreases dramatically for splits with
balance worse than 1:4. Therefore, these methods will likely be under powered when used for genetic markers (i.e. SNPs) with low minor allele frequency. If these criteria are met, then our method is very robust to the number of potential splits, with the power being stable with a large number of covariates (up to 1000 in the simulations). Using a more efficient implementation of the algorithm and parallel computing, models with an even larger number of potential splits should also perform well.

In addition to being scientifically relevant, adjusting for confounders in the splitting rule seems to have statistical benefits. Our simulations have shown that adjusting for a confounder is much more efficient than creating a new split. Indeed, we needed to double our sample size from 500 to 1000 to have similar power when the true tree structure changed from one to two splits, however controlling for 4 confounders only resulted in a slight drop in power. Further research into splitting rules that move more of the modelling into the splits themselves rather than the topology of the tree seems worth pursuing.

In the pruning algorithm the cross-validated goodness of split is calculated as the mean of the goodness-of-split of trees built from a single fold of data.

$$
G\left(T_{, k}\right)=\sum_{j=1}^{5} G\left(T_{j, k}\right) / 5
$$

One must be especially careful about the number of folds, $f$, since the sample size $|L j| \approx n / f$ used to make the tree $T_{i, k}$ must be large enough to support the true tree structure. The number of folds can then be seen as a sparsity parameter with more folds resulting in sparser trees. Similar consideration must also be taken when choosing the number of confounding variables to include in the model. When working with such small sample sizes, along with the usual concerns about the instability of asymptotic results, the vector corresponding to the splits may be deemed computationally singular with the confounding variables, leading to $G(s, h)=0$ and sparser trees.

When selecting the final tree choosing the penalty parameter $\alpha_{c}=4$ corresponds to the 0.05 significance level of a $\chi^{2}$ random variable which is approximately 4 . This choice is reasonable when treatment has only two levels, because the 'honest' LRT statistic of the split will be asymptotically $\chi^{2}$. In general, when we have $k$ levels of treatment the 'honest' LRT statistic of the split will be asymptotically $\chi^{2}$ and the choice of $\alpha_{c}$ could be chosen as the 0.05 significance level of a $\chi^{2}$ random variable.

In addition to the LRT statistic, we also tested using other novel splitting methods such as the Wald test statistic and absolute value of the fitted parameter of an underlying Cox model. All three splitting rules were found to have similar performance, and the LRT statistic was chosen as it is the most generalizable. Indeed the recursive partitioning algorithm we use is not dependent on the particular choice of model, and other likelihood based models could be used. With some slight modifications the idea of splitting by the ratio of two likelihoods could be extended to any situation in which a likelihood space can be defined.

The current adjusted survival tree models are developed based on the Cox models. However, for the real research studies, some time to event outcomes may not fit the proportional hazard assumption.

We suggest applying proportional hazard assumption test on the data before applying the survival tree model. Further extensions of the adjusted survival tree model are in development to deal with other types of time to event outcomes.

## Author's Contributions

WX and RD Bareco-first authors and are equal contributors to the development and implementation of the adjusted survival tree methodology. IB, FM, and GL provided data for the application of the methodology to the randomized clinical trial.

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